

INTERMEDIATE EXAMINATION, 1880.

TRIGONOMETRY—ALGEBRA.

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A.

1. Given $\tan A = \frac{4}{3}$, find $\sec A$ and $\operatorname{versin} A$.

2. In any triangle prove

$$\cos \frac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}$$

3. Assuming the diameter of the earth to be 7.926 miles, calculate the dip of the sea horizon as seen from a mountain 4 miles high.

4. Prove that the expression for any angle in circular measure can be converted into seconds by multiplying it by a constant number, and calculate the number.

5. Define a logarithm, and prove that the logarithm of a quotient of two numbers is equal to the difference of the logarithms of the numbers.

6. What convention is laid down in order to affix a meaning to such expressions as $a^{-\frac{1}{2}}$ and $b^{\frac{2}{3}}$? Adopting it, find what these denote, and find other expressions for them.

7. Solve the equations:—

$$\frac{x}{y} = \frac{a}{n}, \quad x + y = a;$$

$$\frac{ax}{b(x+c)} : \frac{bx}{a(x+c)} = 1.$$

B.

8. Simplify the expression:—

$$\frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}} \div \frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}},$$

first by rationalizing the numerator, and then by rationalizing the denominator: and then add together your two results.

9. Find the values of x and y from the simultaneous equations:

$$3^x = \frac{27}{3^y}, \quad 2^y = 2^x \sqrt{4}.$$

10. Show that the sum of the cubes of any three consecutive numbers is divisible by three times the middle number.

11. Prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

a. Hence deduce the value of $\tan 2A$ and $\tan(A-B)$.

12. A yacht is 5.8 nautical miles from the mouth of a harbor bearing S. b. W.; in order to reach the harbor she is obliged, by reason of a southerly wind, to make two courses, the first E. S. E., the other S. W. b. W., calculate the distance run in each course, and the whole time, the rate of sailing being 7 knots.

C.

13. Find the sine of $1''$.

14. To find the distance of a column of cavalry I ascertain with a micrometer that its vertical height subtends an angle of $4'$; if we assume the height of a mounted soldier to be 8 feet, what is the distance of the column?

15. Solve the equations:

$$\frac{7x+1}{x-1} = \frac{35}{9} \cdot \frac{x+4}{x+2} + 5\frac{1}{9}$$

$$\frac{2}{x - \sqrt{2-x}} + \frac{2}{x - \sqrt{2-x}} = x.$$

16. The plate of a looking-glass is 18 inches by 12, and it is to be framed with a frame of uniform width, whose area is to be equal to that of the glass; find the width of the frame.