The expression X_2 resembles X in having the coefficients of the various powers of x rational; for it is the H. C. M. of X and $\frac{dX}{dx}$. Hence, denoting $\frac{X}{X_2}$ by X_3, we have, from (6) and (7),

$$(x-\phi_1)(x-\phi_2)......(x-\phi_s) = X_3, \quad\text{.............................} \quad (8)$$

where X_3, being the quotient of X by X_2, must have the coefficients of the various powers of x rational. Hence b_1, b_2, &c, may be exhibited as rational expressions. Thus the former of the two points to be proved in the Corollary is established. Next, should $f(p)$ be in a simple form, and should the numbers λ, β, &c., not be all equal to one another, let λ be less than δ, and not greater than any of the others. Then, from (8) and (6), we have, putting X_4 to denote the quotient of X by X_3^{λ},

$$(x-\phi_2)^{\beta-\lambda}......(x-\phi_s)^{\delta-\lambda} = X_4; \quad\text{.............................} \quad (9)$$

X_4 being rational. Should the numbers, $\beta-\lambda$, $\delta-\lambda$, &c., not be all equal to one another, then, exactly as we reduced equation (6) to equation (9), on the left hand side of which no power of $(x-\phi_1)$ appears as a factor, we can reduce equation (9) to an equation bearing the same relation to (9) that (9) bears to (6). And so on, till we arrive at an equation, such as (9), in which the indices, such as, $\beta-\lambda$, &c., are all equal to one another. Let the result obtained when this point is reached be,

$$(x-\phi_a)^{l-h}(x-\phi_c)^{k-h}......(x-\phi_s)^{\delta-h} = X_5.$$

From this, since the numbers, l, k,......, δ, are equal to one another, we get, by continuing the reduction,

$$(x-\phi_a)(x-\phi_c)......(x-\phi_s) = X_6;$$

X_6 being a rational expression: which, since the number of its factors, $x-\phi_a$, $x-\phi_c$, &c., is less than s, and since $f(p)$ is supposed to be in a simple form, is (Cor. 1) impossible. Hence λ, β, &c., in (6), are all equal to one another; and therefore each of the terms, ϕ_1, ϕ_2, ..., ϕ_s, must recur in (1) the same number of times.

Cor. 3. In $f(p)$ let certain surds, y_1, y_2, &c., (in which series of terms, as was pointed out in Def. 7, all the subordinates of any surd mentioned are included), have definite values assigned to them; and