

559. $ABCDE$ is a regular pentagon; join AC and BD intersecting at O : shew that AO is equal to DO , and that the rectangle AC, CO is equal to the square on BC .

560. A straight line PQ of given length moves so that its ends are always on two fixed straight lines CP, CQ ; straight lines from P and Q at right angles to CP and CQ respectively intersect at R ; perpendiculars from P and Q on CQ and CP respectively intersect at S : shew that the loci of R and S are circles having their common centre at C .

561. Right-angled triangles are described on the same hypotenuse: shew that the locus of the centres of the inscribed circles is a quarter of the circumference of a circle of which the common hypotenuse is a chord.

562. On a given straight line AB any triangle ACB is described; the sides AC, BC are bisected and straight lines drawn at right angles to them through the points of bisection to intersect at a point D ; find the locus of D .

563. Construct a triangle, having given its base, one of the angles at the base, and the distance between the centre of the inscribed circle and the centre of the circle touching the base and the sides produced.

564. Describe a circle which shall touch a given straight line at a given point, and bisect the circumference of a given circle.

565. Describe a circle which shall pass through a given point and bisect the circumferences of two given circles.

566. Within a given circle inscribe three equal circles, touching one another and the given circle.

567. If the radius of a circle be cut as in II. 11, the greater segment will be the side of a regular decagon inscribed in the circle.

568. If the radius of a circle be cut as in II. 11, the square on its greater segment, together with the square on the radius, is equal to the square on the side of a regular pentagon inscribed in the circle.

569. From the vertex of a triangle draw a straight line to the base so that the square on the straight line may be equal to the rectangle contained by the segments of the base.

570. Four straight lines are drawn in a plane forming four triangles; shew that the circumscribing circles of these triangles all pass through a common point.