

$$\begin{aligned}
 (17) \quad qn - mr &= y \left\{ -5000k^3y^3 + (-50p_5^3 + 280p_4k^3)y^3 \right. \\
 &\quad + \left(-\frac{2}{5}p_4p_3^3 - 1056k^3p_5 - \frac{248}{25}p_4^3k^3 \right) y \\
 &\quad \left. + k^3 \left(-2048k^4 + \frac{8}{125}p_4^3 + \frac{32}{5}kp_4p_5 \right) \right\}; \\
 rm - 8kq &= y \left\{ 125000y^3 - 3000p_4y^3 + (24p_4^3 + 800kp_5)y \right. \\
 &\quad \left. - \left(-2048k^4 + \frac{8}{125}p_4^3 + \frac{32}{5}kp_4p_5 \right) \right\}.
 \end{aligned}$$

By substituting in (18) these values of $rn - 8kr$, $qn - mr$, $rm - 8kq$, we get

$$F(y) = q_0y^6 + q_1y^5 + \dots + q_5y + q_6 = 0, \quad (19)$$

where

$$\begin{aligned}
 (18) \quad q_0 &= -625000000, \\
 q_1 &= 50000000p_4, \\
 q_2 &= -200000(200kp_5 + 11p_4^3), \\
 q_3 &= 102400000k^4 + 1280000kp_4p_5 + 44800p_4^3, \\
 q_4 &= -(25600kp_4^3p_5 + 4096000k^4p_4 + 640000p_5^3k^3 + 448p_4^4), \\
 q_5 &= 8192k^3p_4p_5^3 - 2048 \times 1856k^5p_5 + 64k^3 + \frac{49152}{5}k^4p_4^3 + \frac{6144}{25}kp_4^3p_5 + \frac{6784}{3125}p_4^5, \\
 q_6 &= -\left(\frac{8}{125}p_4^3 - 2048k^4 + \frac{32}{5}kp_4p_5 \right)^2.
 \end{aligned}$$

§8. Assuming now that the coefficients p_3 , p_4 , p_5 are commensurable quantities, let the commensurable root y of equation (19) be found. Then a^2z is known. Then, from (11) and (16), t or $\frac{c}{a}$ is found.

§9. At this stage, as was indicated in §3, two courses are open to us. One is to proceed to find u_1^5 , u_2^5 , u_3^5 , u_4^5 without troubling ourselves to inquire what a , c , θ , ϕ and h are separately. This, the natural and the shortest course, we will now follow. Since a^2z and $\frac{c}{a}$ are known, their product acz is known. And, by (14), A is known. Therefore $czhe(\theta^2 - \phi^2z)$, which, by the last of equations (7), is equal to $-aczA$, is known. Hence by the first of equations (8), B is known. We might even more simply, acz being known, find B from the second of equations (9). The second of equations (8) gives us

$$y(B\sqrt{z}) = 2c\sqrt{z}(k^2 - c^2z) + a^2zk(a\sqrt{z}) - 2kAa\sqrt{z}. \quad (20)$$

Now acz is known, and it is the same as $(a\sqrt{z})(c\sqrt{z})$; consequently, the signs