

In arithmetic, subtraction should be taught as a shopman gives change because this smooths the way for the improved long division. The highest digit of the multiplier should be used first in the multiplication of whole numbers, because this facilitates the approximate multiplication of decimal fractions. Arithmetic may be taught so as to prepare the student to pay regard to dimensions in mechanics, and in the treatment of circulating decimals either sound ideas may be given, facilitating the acquirement of the important doctrine of limits, or confused ideas may be allowed to be formed retarding the pupil's progress and throwing additional difficulties in his way. It were better not to touch this part of the subject at all than to do this; in fact, in accordance with the first law of teaching, if the subject is beyond the pupil's comprehension the teaching of it should be deferred. The particular instance quoted, however, is quite intelligible at a much younger age than is often supposed.

Mathematics are sometimes thought to require operations of the mind different from those employed in other studies.¹¹ It is said not to encourage the art of observation. As we all know, this is not the case. The teacher should take opportunities of exercising this faculty, and should ask his pupils to notice distinctions, and when they have discussed these distinctions and described them by a clumsy periphrasis, when the *thing* has thus become known, the *name* "binomial," "homogeneous," or whatever it is, may be given: there are many opportunities for lessons in classification based on observed distinctions.

Another common mistake is to suppose that mathematics do not admit of

reasoning from analogy; on the contrary, the good teacher bases his pupil's apprehension of mathematical method upon the analogy which he guides them to observe in the mode of solution of similar problems.

Inductive reasoning is also said not to appear in mathematics. On the contrary, the mathematical teacher will often lead his pupils by particular examples to guess by induction a general formula, and then he will be able to shew them how to verify their guess, and ascertain whether their induction is justified. They will sometimes guess wrong, and thereby learn to distrust an unverified induction.

The use of experiment to ascertain whether a proposed theorem is likely to be true, is another instance of the application of a mode of investigation often thought to be foreign to mathematics.

Another subject most intimately connected with mathematics is language. One of the aspects of algebra and the differential calculus is distinctly linguistic. It is a valuable exercise for the pupil to translate his symbols into the English language. Clear and exact language is requisite in order to state mathematical problems, and the necessity of using it constitutes one of the difficulties of the learner, but it is a difficulty in the overcoming of which mathematics give much of the intellectual benefit which is supposed only to be derived from the study of a foreign language. The power of stating a physical problem in the language of the differential calculus is an acquisition worth having, even if the subsequent solution of the problem is at present impossible.

After the three great laws of teaching, the Law of Understanding, the Law of Sequence, and the Law of Continuity, some minor maxims may be adduced.

¹¹Of observation, experiment, induction, analogy, the mathematician knows nothing." Sir W. Hamilton.