

can be expressed in terms of a, b , and c . Now if C itself (not its numerical value, but the absolute angle) is determined by a, b , and c ; and if, nevertheless, it cannot in the nature of things be expressed in terms of a, b , and c ; Legendre's demonstration, the very foundation of which is, that a quantity which is determined by certain others, can be expressed in terms of them, must be pronounced erroneous.

Should it be maintained that C (the angle itself) may be expressed in terms of the numbers β and γ , *a right angle being understood to be the unit of measure*; or more fully, thus:

$$C = \text{right angle} \times f(\beta, \gamma);$$

I reply that in the same manner the line c , in Legendre's reasoning, may be expressed in terms of A, B, C , *some line L being understood to be the unit of linear measure*; thus:

$$c = L \times f(A, B, C.)$$

I am inclined to believe, from metaphysical considerations, that it is impossible to demonstrate the properties of parallel lines without a special axiom. As it would be difficult, however, to bring out the grounds of this belief without entering into a somewhat lengthened discussion of the nature of our conceptions of geometrical magnitudes, I content myself in the meantime with the above remarks on Legendre's treatment of the subject. Had the reasoning of that distinguished mathematician been valid, it would have been a standing and conclusive refutation of any theory of our conceptions of geometrical magnitudes, in which the impossibility of proving the properties of parallel straight lines without a special axiom was involved. But as Legendre's demonstration, like all others in which the same thing has been attempted, has been shewn to be erroneous, the ground is clear; and a theory of our geometrical conceptions, such as has been referred to, is at least not exposed to the ready-made fatal objection that it is at variance with unquestioned fact.
