## VIII. ANNUITIES AT COMPOUND INTEREST.

Symbols same as before. Then last payment being received as before $= A$, last but one $= A(1+r)$ last but two $= A(1+r)^2$, last but three $= A(1+r)^3$; and so on, hence 1st payment $= A(1+r)^{t-1}$ $\therefore$ $M = A + A(1+r) + A(1+r)^2 + A(1+r)^3 + \dots\dots A(1+r)^{t-1}$; a geometrical progression, whence by (40) $M = \frac{A((1+r)^t - 1)}{r}$ (60); $A = \frac{Mr}{(1+r)^t - 1}$ (61); and $t = \frac{\log.(Mr + A) - \log. A}{\log.(1+r)}$ (62). From (15) $v(1+r) = M = \frac{A((1+r)^t - 1)}{r}$ $\therefore$ $v = \frac{A((1+r)^t - 1)}{r(1+r)^t}$ (63); $A = \frac{vr(1+r)^t}{(1+r)^t - 1}$ (64); and $t = \frac{\log. A - \log.(A - vr)}{\log.(1+r)}$ (65). To find the present value of an annuity which is to commence after $s$ years and continue for $t$ years; from (63) $v$ for $s + t$ years $= \frac{A}{r}\left(\frac{(1+r)^{s+t} - 1}{(1+r)^{s+t}}\right)$; and for $t$ years only $v = \frac{A}{r}\left(\frac{(1+r)^t - 1}{(1+r)^t}\right)$

$\therefore$ for $t$ years to commence after $s$ years, $v = \frac{A}{r}\left(\frac{1}{(1+r)^s} - \frac{1}{(1+r)^{s+t}}\right)$ (66).

When an annuity lasts for ever, as in the case of landed property, $\frac{1}{(1+r)^t}$ in (63) $= \frac{1}{\infty} = 0$; hence for a perpetual annuity $v = \frac{A}{r}$ (67); $A = vr$ (68); $r = \frac{A}{v}$ (69). The present value of a freehold estate to a person to whom it will revert after $s$ years is found from (66) and is represented by $v = \frac{A}{r(1+r)}$ (70).