always subsists. Hence $t_2=1$, $t_3=1-t^2$, &c. The law of the formation of series (3), expressed in equation (4), being the same with that of series (1), expressed in equation (2), the difference between the series (3) and the series (1) arises solely from the difference in their first terms. The general terms T_m and t_m are easily found. In fact, m being any number greater than zero,

$$T_m=1-mt^2+\frac{mt^4}{\lfloor 2}(m-3)-\frac{mt^6}{\lfloor 3}(m-5)(m-4)+\frac{mt^8}{\lfloor 4}(m-7)(m-6)(m-5)-\text{\&c.}, \quad\ldots\ldots\ldots\ldots \quad (5)$$

$$\text{and } t_m=1-(m-2)t^2+\frac{(m-3)(m-4)}{\lfloor 2}t^4-\frac{(m-4)(m-5)(m-6)t^6}{\lfloor 3}+\text{\&c.}, \quad\ldots\ldots\ldots\ldots \quad (6)$$

When m is even, the number of terms in the value of T_m is $\frac{m+2}{2}$, and $\frac{m}{2}$ in the value of t_m. When m is odd, the number of terms in each of the expressions T_m and t_m is $\frac{m+1}{2}$. To prove (5), we observe that $T_1=1$, and $T_2=1-2t^2$. Henee the law is true for the first two steps. Assume it to hold for $m-1$ steps. Then

$$T_{m-1}=1-(m-1)t^2+\frac{(m-1)t^4}{\lfloor 2}(m-3)-\text{\&c.}$$

and $t^2\,T_{m-2}=\qquad t^2-(m-2)t^4+$ &c. Therefore, by (2),

$$T_m\;=1-mt^2+\frac{mt^4}{\lfloor 2}(m-3)-\text{\&c.}:$$

which proves the Law universally. In the very same manner equation (6) can be shewn to hold.

§2. The following formulæ may now be established:

If $2t\cos\theta=1$, and $2t\sin\theta=k$,

then $2t^m\cos m\theta=T_m$, ... (7)

and $2t^m\sin m\theta=kt_m$. .... (8)

§3. To prove (7), we remark, that, by hypothesis, the Law holds for the first step, that is, when $m=1$. Assume it to hold for $m-1$ steps. We have only to shew then that it holds for the succeeding step. Now, since the Law holds for $m-1$ steps,