"The condition of the water column is now what it was just before the gate was closed, except that its velocity, v, has now the opposite direction—i.e., toward the origin.

"(3) The kinetic energy of the water column, moving toward the origin or away from the gate, is now reconverted into potential energy, which manifests itself in an extension of volume of the water to a subnormal pressure beginning with section 1, and concluding only when the entire water column has been reduced to the subnormal pressure.

"During process (3) water continues flowing from the pipe into the reservoir.

"(4) When the subnormal pressure has been established throughout the length of the pipe, and all the water has come to rest, the water from the reservoir will again direct itself into the pipe, restoring the normal pressure, first in section n, next to the reservoir, and then, in rapid succession, in the other sections $(n-1)$,....4, 3, etc., until, when the normal pressure reaches the gate, we have once more the conditions which existed just before the gate was closed—viz., the normal pressure is restored and the water is moving toward the gate with the original velocity, v.

"We have now followed these pulsations of pressure (with the accompanying transformations of energy and flow of water into and back from the pipe) through a complete cycle of four movements, each extending through the length of the pipe. For convenience, we may consider two successive movements of this kind as a 'round trip' through the pipe.

"The gate remaining closed, the whole process is now repeated in a second cycle, which, in turn, is followed by a third, and so on, the amplitude of the pressure vibrations gradually diminishing (because of friction) until the pipe and the water come to a state of rest.

"But although the intensity of the pressure becomes gradually less, the time required for each cycle remains constant for all repetitions.

"This propagation of pressure, consisting of its transmission through all points of the length of the pipe, each point successively repeating the same periodical movements is, in its nature, simply a case of wave motion, like that of a sound wave.

"The velocity of wave propagation is independent of the intensity of the pressure, and depends only upon the properties of the medium through which the propagation takes place—in the case of water-hammer, upon the elasticity of the water and of the pipe."

Mr. Gibson has applied this theory to slow-closing gates by considering that each infinitesimal movement of the gate is instantaneous and produces a small rise of pressure which travels through the penstock in the same manner that a wave produced by total instantaneous closing would travel. These small gate movements occur in succession and produce in turn their pressure waves which travel back and forth through the penstock. An algebraic sum of these waves gives the rise of pressure existing at any time during the closing stroke.

The writer is of the opinion that the pressure existing throughout the length of the pipe for a slow-closing gate varies almost directly from a maximum at the gate to zero at the point of relief, except where the time of closing is less than $2L/a$.

In Fig. 11, the pressure existing at four points on the penstock has been plotted. The magnitude of the individual pressure waves is not to scale, but their values are shown for the particular example taken. The pressure existing at a point $L/2$ from the gate is approximately one-half the pressure existing at the gate. The pressure existing at a point $3L/4$ from the gate is approximately one-quarter of the pressure existing at the gate.

The diagram showing the pressure existing at the point of relief consists of a series of instantaneous rises above normal. On account of the assumption of twelve instantaneous movements of the gate, the pressure at the point of relief must of necessity show twelve instantaneous rises of pressure, as each wave travels undiminished to the point of relief. If twenty-four instantaneous movements were chosen, there would appear twenty-four instantaneous rises of pressure in the same time, but of only about half the magnitude. Hence, in the limit of an infinite number of instantaneous movements of the gate, or uniform slow-closing, the pressure at the point of relief becomes zero. This must be true, because the duration of time of any rise of pressure at the point of relief is zero. Wherever rises of pressure have actually been recorded at the point of relief on a penstock, they are probably due to an instantaneous movement of the gate during the stroke, or to some other cause, such as a sudden elastic deformation of the waterway, which would have the same effect.

In the translation of Professor Joukovsky's work there are shown tables of pressure rises at different points on the pipes for different velocities destroyed. In every case the pressure waves travel practically undiminished almost to the point of relief. The readings taken at the station nearest the point of relief, however, show a reduction in pressure of from 60% to 80%. This is very significant to the writer, as an indication that the pressure travelled undiminished to a certain point and then gradually was reduced to zero at the point of relief. Although Professor Joukovsky experimented with very rapidly closing gates, as nearly instantaneous as possible, obviously a certain fraction of a second must elapse while the gate is closing. When the time of closing is less than one interval, $2L/a$, the resultant pressure will travel undiminished to a point which is $Ta/2$, in feet, from the

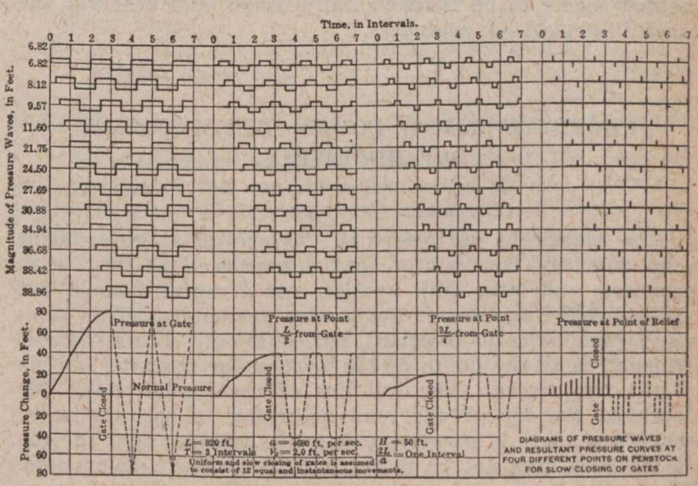

FIG. 11—PRESSURE AT FOUR POINTS ON PENSTOCK

point of relief (where T is equal to the closing time, in seconds). From this point it will gradually be reduced to zero at the point of relief. Granting that some short period of time must elapse during the closing of the gates in Professor Joukovsky's experiments, his actual pressure readings seem to verify the writer's claim.

In the example taken, the duration of closure is assumed to be three intervals. The diagrams and curves have been plotted beyond the three intervals to show the fluctuations of pressure which take place after the gates are closed. Experiments should show that the subsequent fluctuations after the gates are closed are quickly reduced in intensity by internal friction, but this friction should have no effect on the speed of propagation of the pressure waves. Internal friction has of necessity been omitted in these computations, as it is an unknown quantity.

Any other example than that shown by Fig. 11 may be chosen, and the pressure waves may be computed by Mr. Gibson's formulas. A series of diagrams may then be plotted similar to those shown by the writer, and the pressures may be computed for any point on the penstock.

In making the computations for the magnitude of the pressure waves, it is necessary to determine first the number of intervals of time (equal to $2L/a$) contained in the total closing time. The gate movement may then be divided into a number of instantaneous movements equal to the number of intervals. Computations carried out on this basis will give correct points on the pressure-rise curve at the end of each interval. In the event of the closing time being only a few intervals, it is desirable to subdivide the gate movement further in order to develop the curve during the in-