Let ω represent the angular velocity of rectangle = $2\pi n$.

Let A represent the area of the rectangle = ld.

When the rectangle has turned an angle θ from the

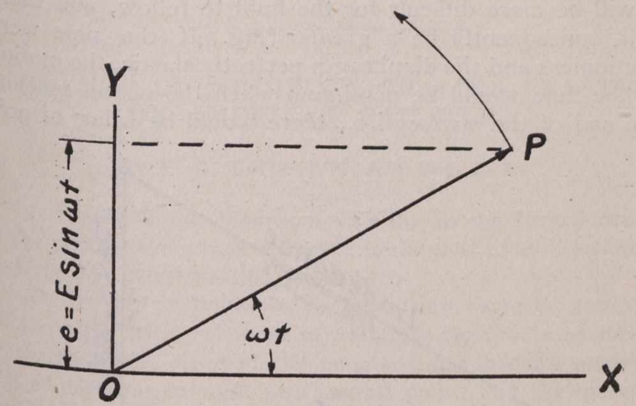

FIG. 51

vertical position, as shown in Fig. 50, the wires forming its sides have a velocity at right angles to the lines of force of V sin θ. The lines of force cut per second by the sides will thus be 2HlV sin θ. But V = πdn, and if time is reckoned from the instant when the rectangle was in the verticle position, $\theta = \omega t$. Since the ends of the rectangle do not cut lines of force, the total instantaneous e.m.f. generated will be

$$e = 2Hld\pi n \sin \omega t = \omega HA \sin \omega t \quad (20)$$

If the rectangle is made up of N turns of wire instead of one turn an e.m.f. will be generated in each turn, and, since these e.m.fs. will act in the same direction around the circuit, the total e.m.f. impressed on the brushes will be

$$e = \omega HAN \sin \omega t \quad (21)$$

The e.m.f. represented by this equation is obviously an alternating one, for the value of sin ωt changes periodically from + 1 to — 1. It will also be observed that the maximum value of the e.m.f. is ω HAN, which is constant. If E is taken to represent this maximum value (positive or negative), equation (21) may be written

$$e = E \sin \omega t \quad (21a)$$

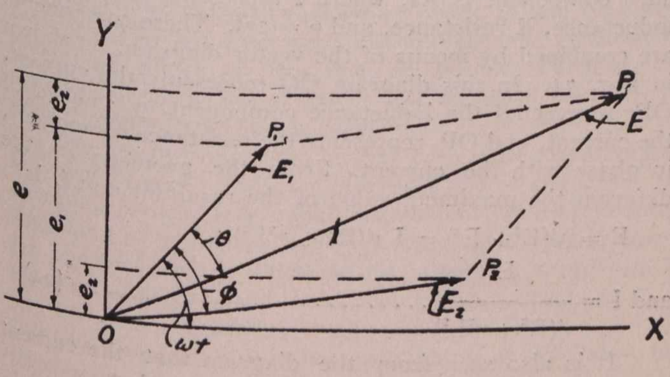

FIG. 52

This equation is a particular case of the general expression x = a sin y, which is known as a simple harmonic function, and applies to many natural phenomena. For example, if velocity is substituted for e.m.f. in equation (20a) the expression will represent a simple harmonic motion. In each case the time in which a complete cycle of changes takes place is known as the "period." The number of cycles or periods per second is known as the "frequency." In the illustration (Fig. 50) the frequency is equal to the number of revolutions = n. At any instant the angle θ through which the coil has turned from the zero-of-time position is known as the "phase." When θ is 90° the e.m.f. is said to be in quarter phase. Any alternating e.m.f. which can be expressed by a simple harmonic function as in equation (21a) is known as a "simple harmonic." Similarly with alternating currents. Alternating e.m.fs. and currents which are not harmonic tend to become harmonic. For this reason and for the reason that the harmonic law lends itself easily to mathematical treatment, designers and builders endeavor to construct alternating current generators so that as far as possible they will generate e.m.fs., which will vary according to this law. All calculations relating to these quantities, except where a high degree of precision is required, are based on the above assumption.

A graph representing an alternating current or e.m.f., plotted on a base of time, as shown in Fig. 49, is usually known as a simple harmonic diagram. For some purposes it is more convenient to represent such quantities by a diagram, as shown in Fig. 51. In this diagram OP is a line drawn to represent the maximum e.m.f., E; OX is a reference axis, and OY is a line

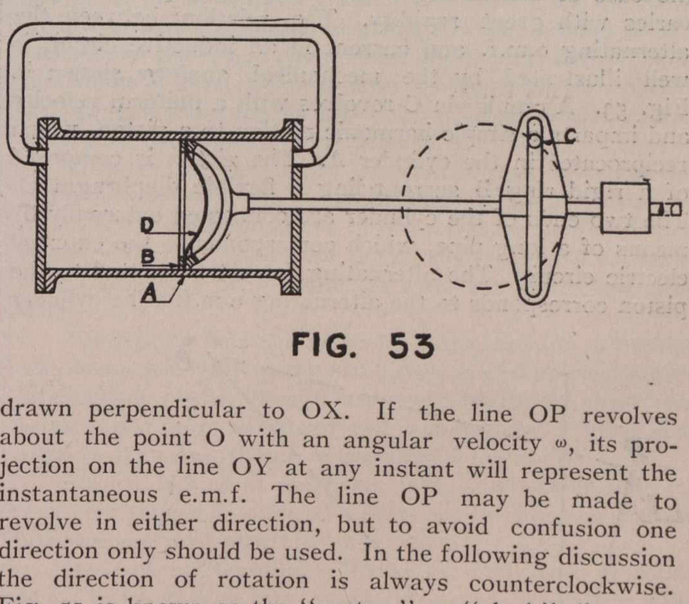

FIG. 53

drawn perpendicular to OX. If the line OP revolves about the point O with an angular velocity ω, its projection on the line OY at any instant will represent the instantaneous e.m.f. The line OP may be made to revolve in either direction, but to avoid confusion one direction only should be used. In the following discussion the direction of rotation is always counterclockwise. Fig. 51 is known as the "vector," or "clock" diagram.

The clock diagram is convenient for use when it is required to find the resultant of two e.m.fs. differing in magnitude and phase. Suppose the two e.m.fs. to be represented by the equations $e_1 = E_1 \sin \omega t$ and $e_2 = E_2 \sin (\omega t - \theta)$. According to these equations the second e.m.f. lags behind the first one by an angle θ; i.e., the angle θ is the difference in their phases at all times. These e.m.fs. are both represented in the clock diagram (Fig. 52). At any instant the total instantaneous e.m.f. is $e = e_1 + e_2$, which is represented by the projection OY of the diagonal of the parallelogram formed on OP_1 and OP_2, representing E_1 and E_2. The resultant of these two harmonic e.m.fs. is, therefore, a simple harmonic e.m.f., of which the maximum value is represented by OP = E.

It is thus seen that alternating e.m.fs. are combined in the same way as mechanical forces. The same is true of alternating currents in parallel.