C Observe that p or q is involved only in a single dimension. Hence arrange with p or q as letter of reference. See McLellan's Algebra, "Factoring by Parts," p. 80.

$\therefore\ p(x^3-x^2+x)-q(x^2-x+1)$
$=(x^2-x+1)(px-q)$

D $=(a^4)^2-(4b^2)^2=(a^4+4b^2)(a^4-4b^2)$
$=(a^4+4b^2)(a^2+2b)(a^2-2b)$. N.B.—It is possible to split the quantity a^4+4b^2 into factors by adding $4a^2b$ and subtracting $4a^2b$ but the coefficients are no longer rational quantities.

6. If $9x^4-30x^3y+Qx^2y^2-10xy^3+y^4$ is a perfect square, find the value of Q.

Answer: $Q=31$.

By inspection, thus: $9x^4$ is the square of $3x^2$; twice the product of this into the next term of the root must give $-30x^3y$, $\therefore$ the second term of the root is $-5xy$. The y^4 is the square root of y^2, and as the $10xy^3$ is $-$ while the $5xy$ is $-$ it is evident that the y^2 must be $+$. Thus the root must be $3x^2-5xy+y^2$, from which it follows by squaring that the coefficient of x^2y^2 must be $25+6$ or 31.

7. If $u=\frac{1}{2}\left(x+\frac{1}{x}\right)$, and $v=\frac{1}{2}\left(y+\frac{1}{y}\right)$, find the value of

$$uv-\sqrt{1-u^2}\,\sqrt{1-v^2}.$$

Since $u=\dfrac{x^2+1}{2x}$, $1+u=\dfrac{(x+1)^2}{2x}$, and $1-u=-\dfrac{(x-1)^2}{2x}$

$\therefore\ 1-u^2=\dfrac{(x^2-1)^2}{4x^2}(-1)$ and $\sqrt{1-u^2}=\dfrac{x^2-1}{2x}\sqrt{-1}$.

And by symmetry $\sqrt{1-v^2}=\dfrac{y^2-1}{2y}\sqrt{-1}$

$\therefore\ \sqrt{1-u^2}\cdot\sqrt{1-v^2}=\frac{1}{4}\left(x-\frac{1}{x}\right)\left(y-\frac{1}{y}\right)(-1)$.

Hence given expression $=\frac{1}{4}\left(x+\frac{1}{x}\right)\left(y+\frac{1}{y}\right)+\frac{1}{4}\left(x-\frac{1}{x}\right)\left(y-\frac{1}{y}\right)$.

Reducing down we get $\frac{1}{2}\left(xy+\dfrac{1}{xy}\right)$ Answer.

8. Solve the equations:—

(1) $\dfrac{x+a}{x-a}-\dfrac{x-a}{x+a}=\dfrac{x+b}{b-x}+\dfrac{x-b}{x+b}$.

(2) $\sqrt{\dfrac{x-2}{x+2}}+\sqrt{\dfrac{x+2}{x-2}}=4$.

(3) $\dfrac{x+a-b}{x-a+b}=\dfrac{a(x+a+5b)}{b(x+5a+b)}$.

(4) $(x+y)(x^2+y^2)=a$; $x^2y+xy^2=b$.

(1) Complete the divisions expressed by the fractions. The quotients cancel. Thus:—

$2a\left\{\dfrac{1}{x-a}+\dfrac{1}{x+a}\right\}=-2b\left\{\dfrac{1}{x-b}+\dfrac{1}{x+b}\right\}$

i.e., $a.\dfrac{2x}{x^2-a^2}=-b.\dfrac{2x}{x^2-b^2}$ $\therefore\ 2x=0$, and $x=0$, one root.

also $\dfrac{a}{x^2-a^2}=\dfrac{-b}{x^2-b^2}$, whence $x^2=ab$, $x=\sqrt{ab}$.

(2) Clear of fractions and we have

$(x-2)+(x+2)=4(x^2-4)^{\frac{1}{2}}$
$x=2(x^2-4)^{\frac{1}{2}}$. Squaring
$3x^2=16$, $\therefore\ x=\pm\frac{4}{3}\sqrt{3}$.

(3) Adding and subtracting numerators and denominators

$\dfrac{x}{a-b}=\dfrac{x(a+b)+10ab+a^2+b^2}{x(a-b)+(a^2-b^2)}$

$\therefore\quad x=\dfrac{x(a+b)+10ab+a^2+b^2}{x+(a+b)}$

Clear of fractions and cancel, and we have $x=\sqrt{a^2+10ab+b^2}$.

(4) Take first equation and add twice second, and we get
$(x+y)^3=a+2b$, or $x+y=(a+2b)^{\frac{1}{3}}$
$\therefore$ from first $x^2+y^2=a\div(a+2b)^{\frac{1}{3}}$
and from second $2xy=2b\div(a+2b)^{\frac{1}{3}}$
$\therefore\ (x-y)^2=(a-2b)\div(a+2b)^{\frac{1}{3}}$
i.e., $x-y=(a-2b)^{\frac{1}{2}}\div(a+2b)^{\frac{1}{6}}$,
But $x+y=(a+2b)^{\frac{1}{3}}$. Whence by addition and subtraction we have the values of x and y.

9. The product of the sum and difference of a number and its reciprocal is equal to $3\frac{3}{4}$, find the number.

Let x and $\frac{1}{x}$ be the number and its reciprocal,

Then $\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)=\frac{15}{4}$
i.e. $4x^4-15x^2-4=0$
$\therefore$ $x^2=4$ or $-\frac{1}{4}$
and $x=\pm 2$, or $\pm\frac{1}{2}\sqrt{-1}$.

10. Simplify $\left(x^{\frac{1}{5}-\frac{1}{7}}\right)^{\frac{2}{3}}\left(x^{\frac{1}{7}-\frac{1}{3}}\right)^{\frac{2}{5}}\left(x^{\frac{1}{3}-\frac{1}{5}}\right)^{\frac{2}{7}}$; and extract the square root of $2x+\sqrt{3x^2-y^2}$.

(*a*) Expression $=\left(x^{\frac{1}{15}-\frac{1}{21}+\frac{1}{35}-\frac{1}{15}+\frac{1}{21}-\frac{1}{35}}\right)^2$
$=(x^0)^2=(1)^2=1$.

(*b*) Observe that $(2x)^2-(3x^2-y^2)=x^2+y^2$, which is not a perfect square. Hence the result will be more complex than the given expression. If the coefficient of $3x^2$ were changed to four we should get y^2 instead of x^2+y^2, and the criterion would hold. We might then put the square root of the expression $=\sqrt{m}+\sqrt{K}$, &c., &c. See Hamblin Smith's Algebra, Canadian Edition, p. 227.

11. Find a number expressed by two digits whose sum is 10; and such that if 1 be taken from its double the remainder will be expressed by the same digits in reversed order.

Let $x=$ units and $y=$ tens.
$\therefore\ x+y=10$ and $10y+x=$ the number.
Also $2(10y+x)-1=10x+y$. Two simple equations from which we get $x=7$, $y=3$ and number $=37$.

CAMBRIDGE ENG.—PREVIOUS EXAMINATIONS.

ALGEBRA. (*Higher.*)

1. Find a formula for the sum of the first n terms of an arithmetical progression of which the first term is a and the common difference b.

The first term of an arithmetical progression is 3 and the third term 9; find the sum of the first 20 terms.

2. Prove that the sum of a geometrical progression of which the first term is a, the common-ratio r, and the last term l, is $(rl-a)\div(r-1)$.

The sum of a geometrical progression, whose common ratio is 3, is 728, and the last term 486; find the first term.

3. The sum of three numbers in A. P. is 21 and their product is 315; find the numbers.

4. Sum to n terms and, when possible, to infinity the following progressions: (*i*) $9+5+1-3-$&c. (*ii*) $4-3+\frac{9}{4}-\frac{27}{16}+$&c. (*iii*) $\frac{1}{3}+\frac{1}{2}+\frac{3}{4}+\frac{9}{8}+$&c.

5. Prove that $(x-y)(1-az-aw+azw)+(y-w)(1-ax-az+axz)=(1-axz)(1-y-w+ayw)-(1-azw)(1-x-y+axy)=(x-w)(1-ay-az+ayz)$.

6. Prove that if $x=a+d$, $y=b+d$, $z=c+d$, then $x^2+y^2+z^2-yz-zx-xy=a^2+b^2+c^2-bc-ca-ab$.

7. Two passengers have together 500lbs. of luggage, and are charged 5*s.* and 5*s.* 10*d.* respectively for the excess above the weight allowed. If the luggage had all belonged to one of them he would have been charged 15*s.* 10*d.* How much luggage is a passenger allowed free of charge?

8. Define a logarithm; and find the logarithms of (*i*) $\sqrt{2}$, (*ii*) 4, (*iii*) 16, (*iv*) 64, to the base 16.

Given that, to base 10, log $2=\cdot 301000$, and log. $3=0\cdot 4771213$, find the logarithms of (*i*) 72, (*ii*) 14·4, (*iii*) ·00015 and (*iv*) $\{3^2 5^4\div\sqrt{2}\}^{\frac{1}{3}}$.

9. Define the characteristic of a logarithm, and state the rules by means of which the characteristic of a logarithm to base 10 of any given number may be written down by inspection. Divide 5·3010300 by 9. What is the integral part of the logarithm of (*i*) 200, (*ii*) $\frac{1}{300}$ to the base 11?

Using the value of log. 2 given in the previous question, determine how many cyphers there are between the decimal point and the first significant figure in $(\frac{1}{2})^{1000}$.

RESULTS.

1. 630. 2. 2. 3. 5, 7, 9. 4. (*i*) $n(11-2n)$; (*ii*) $\frac{16}{7}\{1-(-\frac{3}{4})^n\}$, $\frac{16}{7}$; (*iii*) $\frac{2}{3}\{(\frac{3}{2})^n-1\}$. 7. 120 lbs. 8 (*i*) $\frac{1}{8}$; (*ii*) $\frac{1}{2}$; (*iii*) 1; (*iv*) $\frac{3}{2}$. (*i*) 1·8573326; (*ii*) 1·1583626; (*iii*) $\bar{4}$·1760913; (*iv*) 1·1998692. 9. $\bar{1}$·4778922. (*i*) 3; (*ii*) −2. 301.