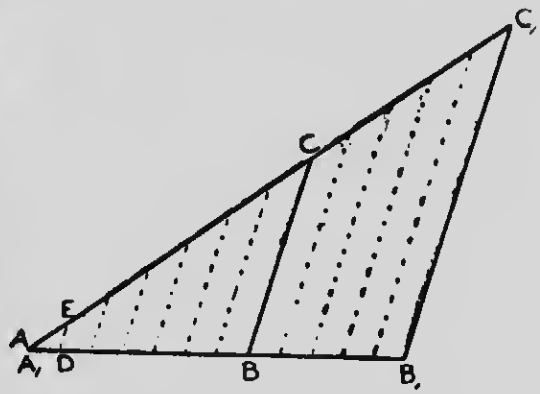
5. The result reached in the preceding article may be demonstrated more generally as follows:

Let **ABC**, $\mathbf{A_1B_1C_1}$ be similar triangles, and let them be placed so that **AB** rests on $\mathbf{A_1B_1}$, and **AC** on $\mathbf{A_1C_1}$, as in the figure. Then **BC** is parallel to $\mathbf{B_1C_1}$. Suppose **AB** and $\mathbf{A_1B_1}$ commensurable, and let **AB** contain n units, and $\mathbf{A_1B_1}$ contain n_1 units. Suppose $\mathbf{A_1B_1}$ divided into its units, and through the points of division draw lines parallel to **BC** or $\mathbf{B_1C_1}$. Evidently the divisions of $\mathbf{A_1C_1}$ are all equal to one another, though not necessarily equal to those of $\mathbf{A_1B_1}$. Then also **AC** contains n parts equal to **AE**, as **AB** contains n parts equal to **AD**; and $\mathbf{A_1C_1}$ contains n_1 parts equal to **AE**, as $\mathbf{A_1B_1}$ contains n_1 parts equal to **AD**. Hence



$$\frac{AB}{A_1B_1} = \frac{n}{n_1} = \frac{AC}{A_1C_1}$$

In like manner the proportionality of the sides about the other equal angles may be shown.

6. On the other hand, if the lengths of the sides of one triangle may be obtained from the lengths of the sides of another by multiplying or dividing each by the same number; that is, if the sides of two triangles, taken in order, are proportional, what relation exists between the angles of the two triangles?