7. If θ be the circular measure of an angle $< \frac{\pi}{2}$, show that the limiting value of $\frac{\sin\theta}{\theta}$, and of $\frac{\tan\theta}{\theta}$, when θ is indefinitely diminished, is 1.

If $f(\theta)$ be a function of θ given by the equation $f(2\theta) = (1-\tan^2\theta)f\theta$, and $f(0)=m$, shew that $f(\theta)=m\,\theta\cot\theta$.

8. If $27r^2$ is less than $4q^3$ shew that the roots of the equation $x^3-qx-r=0$ are

$$2\left(\frac{q}{3}\right)^{\frac{1}{2}}\cos\alpha, \text{ and } 2\left(\frac{q}{3}\right)^{\frac{1}{2}}\cos\left(\frac{2\pi}{3}\pm\alpha\right)$$

$$\text{where } \cos 3\alpha = \frac{r}{2}\left(\frac{3}{q}\right)^{\frac{3}{2}}.$$

9. Shew that $\sin\alpha = \alpha - \frac{\alpha^3}{\underline{|3}} + \frac{\alpha^5}{\underline{|5}} - \frac{\alpha^7}{\underline{|7}} + \ldots\ldots$

$$\cos\alpha = 1 - \frac{\alpha^2}{\underline{|2}} + \frac{\alpha^4}{\underline{|4}} - \ldots\ldots\ldots$$

10. If n is a positive integer, prove that

$$2\cos n\theta = (2\cos\theta)^n - n(2\cos\theta)^{n-2} + \frac{n(n-3)}{1\cdot 2}(2\cos\theta)^{n-4} - \ldots$$
$$\ldots + (-1)^r\frac{n(n-r-1)(n-r-2)\ldots(n-2r+1)}{\underline{|r}}(2\cos\theta)^{n-2r} + \ldots$$

11. Shew that

$$\cos x = \frac{e^{x\sqrt{-1}} + e^{-x\sqrt{-1}}}{2}, \quad \sin x = \frac{e^{x\sqrt{-1}} - e^{-x\sqrt{-1}}}{2\sqrt{-1}}$$

Find the sum to n terms of series.

$$\sin\alpha + c\sin(\alpha+\beta) + c^2\sin(\alpha+2\beta) + \ldots + c^{n-1}\sin\{\alpha+(n-1)\beta\}.$$

12. Resolve $\sin\theta$, and $\cos\theta$ into their factors.

Shew that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots \text{ to infinity} = \frac{\pi^2}{8}$$

$$1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \ldots\ldots \text{ to infinity} = \frac{\pi^4}{96}.$$

ANALYTICAL PLANE GEOMETRY.—HONORS.

Examiners: A. K. Blackadar, B.A.; F. Hayter, B.A.

1. Investigate the equations to a straight line in the following forms:—

$$\frac{x}{a} + \frac{y}{b} = 1, \ (2)\ y = mx + c, \ (3)\ \frac{x-x'}{A} = \frac{y-y'}{B} = r.$$

Shew that the area of the triangle formed by the three straight lines

$$\frac{x}{a} + \frac{y}{b} = 1 = \frac{x}{b} + \frac{y}{a} \text{ and } x+y = \frac{1}{2}(a+b), \text{ is } \frac{1}{8}\,\frac{(a-b)^3}{a+b}$$

2. Find the angle between the two straight lines,

$$Ax+By+C=0 \text{ and } A'x+B'y+C'=0,$$

and deduce the condition of perpendicularity and that of parallelism.

Shew that the two straight lines represented by the equation $ax^2-2(1-b)xy-ay^2=0$, bisect the angles between the two straight lines represented by $bx^2-axy+y^2=0$.

3. Shew how the position of a point is expressed by the method of *polar co-ordinates*.

Find the distance between two points whose polar co-ordinates are given.

4. Write down the general equation of the second degree, and find the co-ordinates of the centre.

What is the condition that the equation may represent (1) central curves, (2) non-central curves, (3) two parallel or coincident straight lines?

If the equation

$$ax^2+by^2+2nxy+2mx+2ly+c=0$$

represents two intersecting straight lines, shew that

$$abc = (l\pm\sqrt{l^2-bc})(m\pm\sqrt{m^2-ca})(n\pm\sqrt{n^2-ab}).$$

5. Obtain the equation of a circle. What does the equation become when the circle is referred to a pair of tangents as axes?

Find the locus of the centre of a circle which passes through a given point and touches a given straight line.

6. Find the equation to the *Radical Axis* of two circles, and shew that if from any point of it straight lines be drawn to touch both circles, the lengths of these lines are equal.

7. Find the tangent and the normal to an ellipse at any point.

If from the point P on an ellipse a normal PUV be drawn meeting the major and minor axes in the points U and V respectively, prove that

$$\frac{PU}{PV} = \frac{b^2}{a^2}.$$

8. If in the ellipse or hyperbola two diameters be such that one of them bisects all chords parallel to the other, then the second will bisect all chords parallel to the first.

If tangents at P and D, ends of conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, meet in the point T, and if (x', y') be the co-ordinates of P, and (x, y) those of T, prove that

$$\frac{x}{a}\left(\frac{x'}{a} + \frac{y'}{a}\right) = \frac{y}{b}\left(\frac{x'}{a} - \frac{y'}{b}\right).$$

9. Prove the following focal properties in an ellipse:—

(1) $SC = eAC$.

(2) $SP+PH = 2AC$.

(3) If SP meets CD in E, shew that $PE = CA$.

The four foci of a complete hyperbola lie on a circle, which also passes through the points where the directrices meet the hyperbola, and circumscribes the rectangle formed by the tangents drawn at the ends of the major and minor axes.

10. Write down the equation to the parabola in its simplest form, and shew that the ellipse ultimately becomes a parabola when one vertex and focus are fixed and the major axis increases without limit.

11. Find the equation to the chord joining the points of contact of two tangents drawn from any point $(x'y')$ to a parabola.

Investigate a method of drawing geometrically a pair of tangents to a parabola from a given point, and prove that the chord joining the points of contact of these tangents is bisected by the diameter through the given point.

PROBLEMS.—HONORS.

Examiners: Charles Carpmael, M.A.; A. K. Blackadar, B.A.; F. Hayter, B.A.

1. Find a point within an isosceles triangle such that its distance from each of the base angles in half its distance from the vertical angle.

2. If an exterior angle of a triangle be bisected by a straight line which likewise cuts the base; the rectangle contained by the sides of the triangle, together with the square on the line bisecting the