geometrical figures, but it also adds new properties which are often useful for the purposes of transformation or of proof. Thus it has recently been shown that in four dimensions a closed material shell could be turned inside out by simple flexure, without either stretching or tearing; and that in such a space it is impossible to tie a knot.

Again, the solution of problems in geometry is often affected by means of algebra; and as three measurements, or co-ordinates as they are called, determine the position of a point in space, so do three letters or measurable quantities serve for the same purpose in the language of algebra. Now, many algebraical problems involving three unknown or variable quantities admit of being generalised so as to give problems involving many such quantities. And as, on the one hand, to every algebraical problem involving unknown quantities or variables by ones, or by twos, or by threes, there corresponds a problem in geometry of one or of two or of three dimensions; so on the other it may be said that to every algebraical problem involving many variables there corresponds a problem in geometry of many dimensions.

There is, however, another aspect under which even ordinary space presents to us a four-fold, or indeed a mani-fold, character. In modern Physics, space is regarded not as a vacuum in which bodies are placed and forces have play, but rather as a plenum with which matter is co-extensive. And from a physical point of view the properties of space are the properties of matter, or of the medium which fills it. Similarly, from a mathematical point of view, space may be regarded as a *locus in quo*, as a plenum, filled with those elements of geometrical magnitude which we take as fundamental. These elements need not always be the same. For different purposes different elements may be chosen; and upon the degree of complexity of the subject of our choice will depend the internal structure or mani-foldness of space.

Thus, beginning with the simplest case, a point may have any singly infinite multitude of positions in a line, which gives a one-fold system of points in a line. The line may revolve in a plane about any one of its points, giving a two-fold system of points in a plane; and the plane may revolve about any one of the lines, giving a three-fold system of points in space.

Suppose, however, that we take a straight line as our element, and conceive space as filled with such lines. This will be the case if we take two planes, *e.g.*, two parallel planes, and join every point in one with every point in the other. Now, the points in a plane form a two-fold system, and it therefore follows that the system of lines is four-fold; in other words, space regarded as a plenum of lines is four-fold. The same result follows from the consideration that the lines in a plane, and the planes through a point, are each two-fold.

Again, if we take a sphere as our element we can through any point as a centre draw a singly infinite number of spheres, but the number of such centres is triply infinite; hence space as a plenum of spheres is four-fold And, generally, space as a plenum of surfaces has a mani-foldness equal to the number of constants required to determine the surface. Although it would be beyond our present purpose to attempt to pursue the subject further, it should not pass unnoticed that the identity in the four-fold character of space, as derived on the one hand from a system of straight lines, and on the other from a system of spheres, is intimately connected with the principles established by Sophus Lie in his researches on the correlation of these figures.

If we take a circle as our element, we can around any point in a plane as a centre draw a singly infinite system of circles; but the number of such centres in a plane is doubly infinite; hence the circle in a plane form a three-fold system, and as the planes in space form a three-fold system, it follows that space as a plenum of circles is six-fold.

Again, if we take a circle as our element, we may regard it as a section either of a sphere, or of a right cone (given except in position) by a plane perpendicular in the axis. In the former case the position of the centre is three-fold; the direction of the plane, like that of a pencil of lines perpendicular thereto, two-fold; and the radius of the sphere one-fold; six-fold in all. In the latter case, the position of the vertex is three-fold; the direction of the axis two-fold; and the distance of the plane of section one-fold; six-fold in all, as before. Hence space as a plenum of circles is six-fold.

Similarly, if we take a conic as our element, we may regard it as a section of a right cone (given except in position) by a plane. If the nature of the conic be defined, the plane of section will be inclined at a fixed angle to the axis; otherwise it will be free to take any inclination whatever. This being so, the position of the vertex will be three-fold; the direction of the axis two-fold; the distance of the plane of section from the vertex one-fold; and the direction of that plane one-fold if the conic be defined, two-fold if it be not defined. Hence, space as a plenum of definite conics will be seven-fold, as a plenum of conics in general eight-fold. And so on for curves of higher degrees.

This is in fact the whole story and mystery of manifold space. It is not seriously regarded as a reality in the same sense as ordinary space; it is a mode of representation, or a method which, having served its purpose, vanishes from the scene.

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Absence from the city prevented us from acknowledging the receipt of solutions of problems in the August number of the Journal—of 1, 2 and 3, by Mr. Armstrong, of Woodham; of 2 and 3, by Mr. Jones, of Brentwood; of 1 and 2, by J. M., of Oshawa; and of 2, by Mr. Shaw, of Kemble.

A solution of 3 not having appeared, we give the following by Mr. Paris, the proposer:

$$\begin{aligned} \tfrac{1}{6} \text{ of 6 lbs.} &= \text{1 lb. at 65 cts.} = \$0.65 \\ \tfrac{4}{5} \text{ of remainder} &= \text{4 lbs. at 70 cts.} = \phantom{\$}2.80 \\ \text{residue} &= \text{1 lb. at 75 cts.} = \phantom{\$}0.75 \\ & \text{6 lbs. sell for } \$4.20 \end{aligned}$$

1 lb. sells for 70 cts. at a gain of 40 p. c. $\therefore$ 1 lb. costs 50 cts. $\frac{16}{23}$ of an oz. is $\frac{1}{23}$ of a lb. Hence loss is $\frac{1}{23}$ of $\frac{1}{2}$ of the tea, or $\frac{1}{46}$ of the tea. Also the gain is the advance of 5 cts. on 70 cent tea, or a gain of $\frac{1}{14}$ of it. $\therefore$ gain $= \frac{1}{14}(\frac{1}{3} - \frac{1}{46}) = \frac{43}{1932}$ of the tea; and loss is $\frac{1}{46}$; $\therefore$ net gain $=$ difference $= \frac{43}{1932} - \frac{1}{46} = \frac{1}{1932}$ of tea. And this gain is $2\frac{4}{5}$ lbs. at 50 cts. $=$ 2 lbs. at 70 cts. (Reducing to 70 cts., because 70 cts. was previously used as the money equivalent of 1 lb.) $\therefore \frac{1}{1932}$ of tea $=$ 2 lbs., or whole number of lbs. is 3864.

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## PROBLEMS FOR SOLUTION.

1. $ABC$ is a triangle, having the angle at $C$ a right angle; the angle at $A$ is bisected by a straight line which meets $BC$ at $D$, and the angle at $B$ is bisected by a straight line which meets $AC$ at $E$. $AD$ and $BE$ intersect *at* $O$: shew that the triangle $AOB$ is half the quadrilateral $ABDE$, using Book I., Euc., only.

J. M., Oshawa.

2. Let it be required to raise a given weight $W$ to a given height $BC$, along an inclined plane $AC$, by means of another given weight $P$, connected with the former by a flexible rope $WCP$, moving over a pulley at $C$. Find the tension of the rope, also the inclination and length of the plane, so that the time of the whole ascent may be the least possible.

G. Shaw, Kemble.

3. $ABC$ is a triangle; prove that the resultant of the forces represented by $2AB$ and $AC$ is represented by $3AD$; $D$ being a point in $CB$ taken at $\frac{2}{3}$ the way from $C$ to $B$.

R. R. Cochrane, Ottawa.

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J. M.—You are right,—the problem is not correct.

"Latitudinarian."—Your solution is correct.

# Practical Department.

## CONVERSATIONAL COLUMN.

The Editor of the Practical Department will be glad to send forms of application and other information to those teachers and others who desire to become members of the Chautauqua Literary and Scientific Circle, explained in the last number of the Journal.

*I was taught that a concrete number should never be used as the multiplier of an abstract number. For instance, in finding the price of oranges at 3 cents each, I would not regard it as correct for my pupils to multiply the 7 by 3, but 'vice versa.' Now, I notice that in the new and very valuable Elementary Arithmetic published by Messrs. Kirkland and Scott, they have in many instances fallen into the error of indicating that an abstract number should be multiplied by a concrete number.*

Hamblin Smith, in his definition of the sign of multiplication, says that it implies that the second of the two numbers is to be multiplied by the first, Art. 23. His mode of expressing 4 times 67 is $4 \times 67$; $\times$ = times. As the Elementary Arithmetic is intended to be an introductory text-book to H. Smith's Arithmetic, the authors adopted his definition of this sign, Art. 39, and hence such