VIII. If p be nearly equal to q, $\frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}$ is a close approximation to $\left(\frac{p}{q}\right)^{\frac{1}{n}}$; and if $\frac{p}{q}$ differ from 1 only in the $(r+1)^{th}$ decimal place, this approximation will be correct to $2r$ places.

IX. Having given

$$yz+\frac{1}{yz}-ax-\frac{b}{x}=zx+\frac{1}{zx}-ay-\frac{b}{y}=$$
$$yx+\frac{1}{xy}-az-\frac{b}{z}$$

prove that if x, y, z be all unequal, $ab=1$, and each member of these equations $=o$.

10. If $\frac{ax-by}{z}=\frac{ay-bz}{x}=\frac{az-bx}{y}$ prove that $x=y=z$.

11. Prove that

$$3^n=1+n\cdot2^n+\frac{n(n-1)(n-2)}{1\cdot2\cdot3}2^{n-2}$$
$$+\frac{n(n-1)(n-2)(n-3)(n-4)}{1\cdot2\cdot3\cdot4\cdot5}2^{n-4}+\text{etc.}$$

12. If A, B, C are the angles of a triangle, then $\sin(A-B)\sin C+\sin(B-C)\sin A+\sin(C-A)\sin B=0$.

13. If $\sin l=\frac{a-b}{a+b}$, $\sin m=\frac{b-c}{b+c}$, $\sin n=\frac{c-a}{c+a}$, prove that $\sec^2 l+\sec^2 m+\sec^2 n=2\sec l\cdot\sec m\cdot\sec n+1$.

14. A started from Ottawa at 9 A.M., to walk to Chelsea. After he had walked 1⅙ miles, B started and overtook A half-way there. A then increased his pace one-fifth and B decreased his one-ninth, and they reached Chelsea together at 11:28½ A.M. Find the distance to Chelsea.

15. O is the point in AO perpendicular to the straight line ABC, from which BC *appears* the longest; prove that

$$\tan COB=\frac{BC}{2\,AO}.$$

XVI. An object is observed at three points A, B, C, lying in a horizontal straight line which passes directly underneath the object; the angles of elevation at A, B, C are $m, 2m, 3m$, and $AB=a$, $BC=b$; prove that the height of the object is

$$\frac{a}{2b}\sqrt{(a+b)(3b-a)}$$

Solutions to Problems.

VI. Let a, c, be the first and last terms respectively, then

$$a_r=a+(r-1)\frac{c-a}{n-1},\quad b_r=a\left(\frac{c}{a}\right)^{\frac{r-1}{n-1}},$$
$$c_r=\frac{ac(n-1)}{c(n-1)+(r-1)(a-c)}.$$

Required to show

$$\frac{a+r\frac{c-a}{n-1}}{a\left(\frac{c}{a}\right)^{\frac{r}{n-1}}}=\frac{a\left(\frac{c}{a}\right)^{\frac{n-r-1}{n-1}}}{\frac{ac(n-1)}{c(n-1)+(n-r-1)(a-c)}}$$

or
$$\frac{\frac{a(n-1)+r(c-a)}{n-1}}{a\left(\frac{c}{a}\right)^{\frac{r}{n-1}}}=\frac{a\left(\frac{c}{a}\right)^{\frac{n-r-1}{n-1}}}{\frac{ac(n-1)}{a(n-1)+r(c-a)}}$$

or $ac=ac$.

VII. First take 2 cards in each suit, and suppose the cards arranged thus, then 1 in (1) may be taken with 2 in (1), (2) ... (p), thus giving p sets. Now take 1 in (2) and so on for each of the others; thus on the whole there are p^2 sets. Then take 3 cards in each suit; now the p 3's may be arranged with the p^2 sets in the same manner as before, giving p^3 sets, and so on. q cards and p sets taken as in question will give pq sets.

1	2	(1).
1	2	(2).
1	2	(3).
:		:
:		:
1	2	(p).

VIII. Let $\sqrt[n]{\frac{p}{q}}=1+x$ where x is very small,

$$\therefore\ \frac{p}{q}=1+nx \text{ nearly.}$$
$$\therefore\ x=\frac{\frac{p}{q}-1}{n}$$

also $\frac{p}{q}-1=nx+\frac{n(n-1)}{\lfloor 2}x^2$ nearly.