Mean error of a distance R of 100 yards measured by means of the relation:

$$R = \frac{\overline{ab}}{0{\cdot}01} = 100\ \overline{ab}, \textit{ with both instruments.}$$

This measurement necessitates 1 pointing, 1 reading and hence 2 contacts, therefore:

For tacheometer No. 1 the total error E_1, of the rod interval $\overline{ab}$ determined at a distance of 100 yards is:

$$E_1 = \sqrt{1 \times (0{\cdot}25)^2 + 1 \times (0{\cdot}5)^2 + 2 \times (0{\cdot}357)^2} = \sqrt{0{\cdot}0625 + 0{\cdot}25 + 0{\cdot}2549} =$$
$$= \sqrt{0{\cdot}5674} = 0{\cdot}7533.$$

and for tacheometer No. 115:

$$E_{115} = \sqrt{1 \times (0{\cdot}25)^2 + 1 \times (0{\cdot}4)^2 + 2 \times (0{\cdot}225)^2} = \sqrt{0{\cdot}0625 + 0{\cdot}16 + 0{\cdot}1012} =$$
$$= \sqrt{0{\cdot}3237} = 0{\cdot}5687.$$

Here the corresponding errors on the distance are:

$$E_{R_1} = 0{\cdot}7533 \times 100 = 75{\cdot}33 \text{ or } 0{\cdot}07533 \text{ yd. and}$$

$$E_{R_{115}} = 0{\cdot}5687 \times 100 = 56{\cdot}87 \text{ or } 0{\cdot}05687 \text{ yd.}$$

Mean error of a distance R of 100 yards measured by means of the relations:

$$R = \frac{\overline{ab} + \overline{bc} + \overline{bd}}{0{\cdot}03} = \frac{100}{3}(\overline{ab} + \overline{bc} + \overline{bd}) \text{ for tacheometer No. 1 and}$$

$$R = \frac{\overline{ab} + \overline{bc} + \overline{bd}}{0{\cdot}025} = \frac{100}{2{\cdot}5}(\overline{ab} + \overline{bc} + \overline{bd}) \text{ for tacheometer No. 115.}$$

When carrying on levelling operations, it may be found convenient to use this relation, with the intervals counted from a single pointing made near the centre of the rod with the lever abutted against pin b.

In this case we have, therefore, as in the first, 1 pointing, 3 readings and 4 contacts; but the error of pointing, Ep, modifies the adjoining intervals $\overline{ba}$ and $\overline{bc}$ on each side in opposite directions, so that an error on $\overline{ba}$ is neutralized by an equal and opposite error on $\overline{bc}$, and the only interval affected by Ep is $\overline{bd}$. Hence:

$$E = \sqrt{Ep^2 + 3(Er)^2 + 4(Ec)^2}$$

and replacing the symbols by their numerical values, we find for the total error of intervals measured with tacheometer No. 1:

$$E_1 = \sqrt{(0{\cdot}25)^2 + 3(0{\cdot}5)^2 + 4(0{\cdot}357)^2} = \sqrt{0{\cdot}0625 + 0{\cdot}075 + 0{\cdot}5098} = \sqrt{1{\cdot}3225} = 1{\cdot}15,$$

and for the total error of those measured with tacheometer No. 115: