For, $A - a = (b - B)x + (c - C)x^2 + \ldots.$

But the second member changes value as x changes its value, while the first member is constant.

Hence there cannot be equality unless each member is equal to zero. $\therefore$ $A - a = 0$ or $A = a$, and by rejecting A and a as being equal and dividing by x we obtain in like manner $B - b = 0$, or $B = b$, &c. . . . .

The coefficients A, a, B, b, &c., are called *indeterminate* or undetermined coefficients, and the proposition now proved states the principle of indeterminate coefficients.

The principle of indeterminate coefficients is one of the most prolific in algebraic analysis. Some of its simpler application will be illustrated by a few examples.

Ex. 171. To expand $\dfrac{1+x}{(1-x)^2}$ into a series according to ascending powers of x.

$$\text{Put } \frac{1+x}{(1-x)^2} = a + bx + cx^2 + dx^3 + \ldots.$$

$$\text{then } 1 + x = (1 - 2x + x^2)(a + b + cx^2 + dx^3 + \ldots.)$$

$$= a \quad \begin{array}{r|r|r|l} +b & x+c & x^2+d & x^3 \ldots. \\ -2a & -2b & -2c & \\ & +a & +b & \end{array}$$

and equating coefficients of like powers of x,

$$a = 1\,;\ b - 2a = 1 \ \therefore\ b = 3,$$
$$c - 2b + a = 0 \ \therefore\ c = 2b - a = 5,$$
$$d - 2c + b = 0 \ \therefore\ d = 2c - b = 7,$$
$$\&c., \qquad \&c.$$

$$\therefore \frac{1+x}{(1-x)^2} = 1 + 3x + 5x^2 + 7x^3 + \ldots.$$

Compare this result with Ex. 22.

Ex. 172. To expand the square root of $1 + x + x^2$.

$$\text{Put } \sqrt{1 + x + x^2} = a + bx + cx^2 + dx^3 + \ldots.$$

$$\text{Squaring, } 1 + x + x^2 = a^2 + 2abx + \begin{array}{r|r|l} 2ac & x^2 + 2ad & x^3 \ldots. \\ b^2 & 2bc & \end{array}$$