QUESTION DEPARTMENT.

S. C. R. (1) If one angle of a triangle is one third of two right angles, show that the square on the opposite side is less than the sum of the squares on the side forming that angle, by the rectangle contained by these two sides.

Let ABC be the triangle, having the angle ABC equal to one third of two right angles. From A draw AX perpendicular to BC. By Ex. 4 page 59 (Hall & Stevens) it may be easily shown that AB = 2 . BX.

Then by II. 13 $AC^2 = AB^2 + BC^2 - 2\ BX\ .\ BC$
$= AB^2 + BC^2 - AB\ .\ BC.$

(2) Find the locus of a point which moves so that the sum of its distances from two given intersecting straight lines of unlimited length is constant.

Let OA, OB be the two intersecting straight lines, and K the given constant length. At O draw OC perpendicular to OA and equal to K; draw CD parallel to OA meeting OB in D. In OA make OE equal to OD and join DE. Then DE is the required locus; for by Ex. 22, page 99, the sum of the perpendiculars on OA and OB from any point in DE is equal to the perpendicular from D on OA = OC = K.

(3) The area of a quadrilateral is equal to the area of a triangle having two of its sides equal to the diagonals of the given figure, and the included angle equal to either of the angles between the diagonals.

Draw BO parallel to the diagonal AC and CO parallel to AB, then ABOC is a parallelogram. Also the perpendicular from D on BO is equal to the sum of perpendiculars from D on AC and perpendicular from B on AC.

Therefore, triangle DBO = triangles DAC + ABC, since these triangles have equal bases (Ex. 16 (1) Hall & Stevens.)

(4) If a straight line is drawn through one of the angles of an equilateral triangle to meet the opposite side produced, so that the rectangle contained by the segments of the base is equal to the square on the side of the triangle, show that the square on the line so drawn is double of the square on a side of the triangle.

If ABC be an equilateral triangle and if BC be produced to Q so that the rectangle contained by the segments BQ, QC = AC^2 then it is required to prove that $AQ^2 = 2AC^2$

Draw AP perpendicular to BC.

Then $BQ\ .\ QC + PC^2 = PQ^2$ (II 6)
$BQ\ .\ QC + PC^2 + AP^2 = PQ^2 + AP^2$
$BQ\ .\ QC + AC^2 = AQ^2$

But $BQ\ .\ QC = AC^2$ (Hyp.)

Therefore $2AC^2 = AQ^2$

(5) In a right angled triangle, if a perpendicular be drawn from the right angle to the hypotenuse, the square on either side forming the right angle is equal to the rectangle contained by the hypotenuse and the segment of it adjacent to that side.

Draw BQ perpendicular to AC

Then $2AC\ .\ CQ + AQ^2 = AC^2 + CQ^2$

But $AC^2 = AB^2 + BC^2 = 2BQ^2 + AQ^2 + QC^2$ (I 47)

Therefore $2AC\ .\ CQ + AQ^2 = 2BQ^2 + 2CQ^2 + AQ^2$

.. $AC\ .\ CQ = BQ^2 + CQ^2$

.. $AC\ .\ CQ = BC^2$

(6) Coffee it bought at 1s. and chicory at 3d. per lb. In what proportion must they be mixed that 10 per cent may be gained by selling the mixture at 11d. a lb.

11d. = 110 of cost.
10d. = cost.

The mixture is therefore worth 10d. per lb.

Gain on 1 lb. at 12d. = 2d.
.. ½ .. .. .. = 1d.
.. 1 .. .. 3d. = 7d.
.. $\frac{1}{7}$ .. .. .. = 1d.

For every ½ lb. at 12d. there would be $\frac{1}{7}$ lbs. at 3d.
.. .. 7 .. 12d. .. .. 2 .. ..

(7) Solve $19x = 15 - 8x^2$

Transpose, $8x^2 + 19x = 15$

Divide by 8, $x^2 + \frac{19}{8}x = \frac{15}{8}$

Add to both sides $(\frac{19}{16})^2$ $x^2 + \frac{19}{8}x + \frac{361}{256} = \frac{15}{8} + \frac{361}{256}$

Simplify $x^2 + \frac{19}{8}x + \frac{361}{256} = \frac{841}{256}$

Extract square root $x + \frac{19}{16} = \pm \frac{29}{16}$

$x = -\frac{48}{16}$ or $\frac{10}{16}$

$= -3$ or $\frac{5}{8}$

TEACHER. (1)—A man having lent $1000 at 5% interest payable half-yearly, wishes to receive his interest in equal portions monthly, and in advance; how much ought he to receive every month?

Interest to be received each half year = $250.

Interest on $1 for 1 month = $\$\frac{1}{240}$

$\text{Sum} \times (1\frac{6}{240} + 1\frac{5}{240} + 1\frac{4}{240} + 1\frac{3}{240} + 1\frac{2}{240} + 1\frac{1}{240}) = \$250$

Therefore $\text{sum} \times \$(6\frac{21}{240}) = 250$

Therefore $\text{sum} = \$\frac{250}{6\frac{21}{240}}$

$= \$41\frac{33}{487}$

(2) How many pounds of sugar at 8, 13, and 14 cents per pound may be mixed with 3 pounds at 9¼ cents, 2 pounds at 8½ cents, and 4 pounds at 14 cents, so as to gain 16 per cent by selling the mixture at 14½ cents per pound.

Cost price $\frac{100}{116}$ of 14½ cents = 12½ cts.

Diff.	12½				
4½	1 lb. at	8	1		= 22¼ gain.
9¾	3	9¼	1		
8	2	8½	1		
½	1	13	8½	= 4¼	= 22¼ loss
1½	1	14	8	= 12	
6	4	14	1	= 6	

We have therefore 1 lb. at 8 cents.
8½ .. 13 ..
and 8 .. 14 ..

There may be many answers.