fering waves referred to in the foregoing paragraph are then always one interval apart.

After the eighth movement of the gate has taken place, it is found that the next excess pressure has reached 114.00 ft. At the instant the ninth movement takes place it must be noted that the pressure wave produced by the first movement of the gate, which has traveled, during the first interval, from the gate to the origin and back as a wave of super-normal pressure, and during the second interval over the same course as a wave of sub-normal pressure, has now returned to the gate and, again becoming super-normal, is again reflected and commences its journey from gate to origin and back. Thus, at the ninth movement of the gate, there must be taken into consideration the pressure wave caused by the instantaneous destruction of the velocity at that instant, the sub-normal wave due to the fifth movement, and the super-normal wave due to the first movement. After adding to the excess pressure existing at the end of the eighth movement, the rise of pressure caused by the ninth movement, there must be subtracted twice the pressure caused by the fifth movement, and there must be added twice the pressure caused by the first movement, or, in other words, there is subtracted twice the difference between the fifth and first waves.

By thus keeping in mind the position of the wave propagated by each movement of the gate, Table 1 may be completed, and the resulting maximum rise of pressure is found to be 217.70 ft.

TABLE 3—BY ARITHMETIC INTEGRATION

For Data, see Table 1.

(1) Interval.	(2) Time, T.	(3) Gate, B.	(4) Head, H.	(5) Velocity, V	(6) 145 ΔV, Δh.	(7) $\Sigma(\Delta h)$, h_t.
0	0.0	0.91476	165.00	11.75		.....
				0.392	56.8	56.8
1	0.35	0.762	221.8	11.358		
				1.178	171.0	114.2
2	0.70	0.608	279.2	10.18		
				1.91	277.0	162.8
3	1.05	0.457	327.8	8.27		
				2.48	360.0	197.2
4	1.40	0.304	362.20	5.79		
				2.825	410.0	212.8
5	1.75	0.1525	377.8	2.965		
				2.965	430.0	217.2
6	2.10	0.0	382.2	0.00		

Fig. 2 is a series of graphical diagrams showing the magnitude of the pressure waves caused by the successive instantaneous movements of the gate. A separate diagram is drawn for each movement, pressure being represented by the ordinates and time by the abscissas. The change from super-normal to sub-normal and *vice versa* is shown at the end of each interval of time. Fig. 1 is the excess pressure-time curve plotted from the figures in Column 7 of Table 1, or, what amounts to the same thing, from the algebraic sum of the pressure waves shown in Fig. 2.

Table 2 shows the method of determining the results when the recovery of friction in the penstock is included, an example being selected where the friction head is an appreciable quantity. The operations are almost identical with those just described, up to Columns 6 and 7. To avoid reference to the foregoing, however, the formation of Table 2 will be described from the beginning. In the first line, opposite $B = 0.4241$, set down Columns 3, 4, and 7, the known values of H_0, V_0, and h_f (friction head). Next, assume a trial reduction in velocity, caused by the initial instantaneous movement of the gate, and set the figures down in Column 4 under the value of V_0, and subtract it from V_0, placing the difference immediately underneath. This trial figure is ΔV, and is assumed to be destroyed instantaneously by the first movement of the gate. A pressure wave, Δh, of magnitude $= (\Delta V)a/g = (\Delta V)3647/32.2 = 113\Delta V$, therefore, is started up the pipe. The product of 113 ΔV is set down in Column 5 opposite ΔV. In Column 6 is recorded the algebraic sum of the values of Δh. In Column 7 the total friction head, due to the velocity shown in Column 4, is set down, and, in Column 8, the friction head recovered at each operation is shown. For convenience, h_f may be made equal to FV^2, in which F is a coefficient obtained from the known values at the beginning. In the example, $F = 0.3574$. Column 9 shows the sum of the opposite items in Columns 6 and 8. Having obtained the figure in Column 9, it is added to the net head, $H_0 =$ 1,260, and the sum is set down in the next lower line in Column 3. The result must now be checked, to see that $B(H)^{1/2} = V$, where B is now 0.4168, and H and V are the values opposite. If the relation is not satisfied, a new trial value of ΔV must be chosen, and the operations repeated until a check is obtained. After trial, the initial value of ΔV was found to be 0.155. The operations for obtaining the figures on the third line are not quite the same as those just described because, after assuming the next trial value of ΔV, and multiplying it by 113, it must be remembered that the resulting pressure at the gate is reduced by the return of the first wave, which has traveled up to the forebay and back, and now changes to sub-normal and repeats the journey. The time at which the gate is given its second movement has been selected purposely to coincide with the return of the first wave. The item opposite Interval 2, in Column 6, therefore, is the difference between the first two figures in Column 5. The other operations are similar to those already described, and the resulting figures in Columns 3 and 4 are checked similarly with the value of $B = 0.4084$. The succeeding lines are filled in by the same process, always keeping in mind the return of the preceding waves, and whether they change to sub-normal or super-normal. A little study will disclose the fact that the figure in the second line of Column 6 may be obtained by subtracting the figure in the first line of Column 6 from the figure in the second line of Column 5. Similarly, the figure in the third line of Column 6 may be obtained by subtracting the figure in the second line of Column 6 from the figure in the third line of Column 5, and so on.

At this point it is interesting to repeat the trial-and-error work for the foregoing example worked out in Table 1, but using only 6 instantaneous movements of the gate instead of 24. The results are shown in Table 3, and the pressure-time curve in dotted lines in Fig. 1. From these it will be noted that the total rise of pressure is the same as that obtained by the calculations for 24 movements, and, moreover, the resulting pressure at the end of each interval is the same in both cases. Although six movements of the gate, or one movement to each interval of time, are sufficient to determine the pressure rise at the end of each interval, it requires the larger number of movements to obtain intermediate points on the pressure-time curve. If a still greater number of movements is taken, the increments in pressure rise become smaller, and, in the limit, the stepped diagrams similar to Fig. 1 would become a series of smooth curves from the beginning to the end of each interval. The diagram, however, would not necessarily form a smooth curve from the beginning to the end of the closing time, because cusps or changes of curvature at the end of each interval result from the action of the pressure waves in changing at that instant from super-normal to sub-normal or *vice versa*. When the duration of closure is short, the change of curvature in the diagram at the end of each interval is frequently very apparent; but, when the duration of closure is long, the changes of curvature in many cases cannot be detected by the eye. These changes of curvature at the end of the intervals make it difficult to formulate the integration which can so easily be performed by the trial-and-error work already explained.

It is possible, however, to obtain a series of equations, one for each interval of the closing time, that constitute a direct mathematical solution of the problem, without recourse to trial-and-error methods. The foregoing example has been explained in order that the analytical work may be more easily understood, and in order that the method of tracing the course of the pressure waves and keeping track of their periodic changes may be kept clearly in mind. More particularly, it should be borne in mind that though a formula may be written for the pressure rise in the first interval, using only the known quantities existing before the shut-down, the formulas for the pressure rise in any succeeding interval will involve, as one of the known quantities, the value of the pressure rise at the end of the preceding interval. It thus becomes necessary, when calculating the