

conform to proper regulations? We have no doubt about the true answer. "The majority obey because it is for their interest to obey." That is, we suppose, because they want to do so. Very well, this is best; but it is for their interest to be made to obey, if they don't want to.

"The school-room is opened for the assistance of those who want to learn." And for no others, we suppose, by fair inference. This is fine, "new" doctrine. Ask each little rebellious gamin if he wants to learn; if he says he does not, excuse him,—the school-room is not opened for him. "When it appears that a pupil does not want to study, and his conduct is an impediment, he should be excluded until he can become loyal." That is comforting doctrine for lazy, wilful young rebels,—set yourself up against the restraints of the school-room, and you can go and follow your course to destruction at your own sweet will. We think we see him "becoming loyal" in just this way! This is the "Gospel of go-as-you-please" run to seed!

"We haven't much confidence in shingles or whips, or the everlasting 'must.'" There, reader, you have the whole of it. Perhaps "the game was not worth the candle," but there is much similar foolishness all at that it seems necessary to pay some attention to it, occasionally. There is false doctrine enough here to form a foundation for a whole structure of socialism or anarchy. The sooner young rebels and old, and we would not exclude our editor—learn that it is best to come into harmony with the "everlasting must," the better it will be for them and for us all. If they can be brought into this harmony by gentle means, "moral suasion," it is well; but if they cannot, it is wise and kind and "humane" to compel them to conform by force.—B. C. Hewitt in *Illinois School Journal*.

Examination Questions.

UNIVERSITY OF TORONTO.—ANNUAL EXAMINATIONS, 1884.

JUNIOR MATRICULATION.

ALGEBRA.—HONORS.

Examiner: EDOAR FRISBY, M.A.

- Find the continued product of $(x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4)(x^8-x^4y^4+y^8)$
- State and prove Horner's method of Synthetic Division. Apply this method to find the value of $x^8-7x^7+16x^6-3x^5-9x^4+13x^3+4x^2-7x-1800$, when $x=3$.
- Find the Highest Common Divisor of $2x^4+x^3-20x^2-7x+24$ and $2x^4+3x^3-13x^2-7x+15$.
- Find the continued product of the following quantities:
 $x-a\sqrt{-1}$; $x+a\sqrt{-1}$; $x+\frac{a}{2}(\sqrt{3}+\sqrt{-1})$;
 $x+\frac{a}{2}(\sqrt{3}-\sqrt{-1})$;
 $x-\frac{a}{2}(\sqrt{3}+\sqrt{-1})$ and $x-\frac{a}{2}(\sqrt{3}-\sqrt{-1})$
 and prove that
 $\sqrt{4+3\sqrt{-20}}+\sqrt{4-3\sqrt{-20}}=6$.
- Solve the equations
 (1) $x^2-7=\sqrt{x^2-42x+89}$
 (2) $x^2+xy+\frac{y^2}{x^2y}=208$
 $y^2+y\frac{y^2}{x^2y}=1053$.
- State the laws governing the reduction of inequalities and prove that
 $abc > (a+b-c)(b+c-a)(c+a-b) < \left(\frac{a+b+c}{3}\right)^3$
 a, b and c being any positive numbers whatever.

7. Find the limiting values of $\frac{x^2+ax+b}{x^2+cx+d}$.

8. Find the limit of the sum of a geometrical series whose first term is given, the common ratio being less than unity.

The first term of a geometrical series is $\frac{1}{3}$, and the common ratio is $\frac{1}{2}$, find the limit of the sum of the series

9. Find the number of permutations of n letters, of which p are a 's; q are b 's; r are c 's, &c.

How many different permutations can be made of the letters in the word mammalia taken all together?

10. Write down the expansion of $(1+x)^n$

and deduce that of $\frac{1}{\sqrt{1-x^2}}$

and prove that

$$1+3\left(\frac{2n+1}{2n-1}\right)+5\left(\frac{2n+1}{2n-1}\right)^2+\dots+(2n-1)\left(\frac{2n+1}{2n-1}\right)^{n-1}=n(2n-1)$$

n being an integer.

11. Find the greatest term in the expansion of $(1+x)^n$ whenever possible.

What is the number and magnitude of the greatest term in the expansion of $(1-x)^{-6}$ when $x=\frac{1}{2}$.

SOLUTIONS.

1. Ans. $x^{16}+x^8y^8+y^8$. The product of the first pair of factors is seen to be $x^4+x^2y^2+y^4$. Thus the final product is reached by inspection alone.

2. (a) For the neatest method see *Teachers' Handbook*, p. 4.

$$\begin{array}{r} 1-7+16-3-9+13+4-7-1800 \\ +3 \quad +3-12+12+27+54+201+615+1824 \\ \hline 1-4+4+9+18+67+205+608+24 \\ \text{Value}=24 \text{ when } x=3. \end{array}$$

$$\begin{array}{r} 3, \quad A = 2+1-20-7+24 \\ \quad B = 2+3-13-7+15 \\ \quad C = \quad 2+7+0-9 \end{array}$$

$$\begin{array}{r} \text{Again } 5A = 10+5-100-35+120 \\ \quad 8B = 16+24-104-56+120 \\ \quad D = 6+19-4-21 \\ \quad 3C = 6+21+0-27 \end{array}$$

$$\begin{array}{r} 2+4-6. \text{ Strike out C. F. 2 and} \\ E = 1+2-3 \end{array}$$

$$\begin{array}{r} \text{Again } 3D = 18+57-12-63 \\ \quad 7C = 14+49+0-63 \end{array}$$

$$\begin{array}{r} 4+8-12 \\ \therefore F = 1+2-3 = E \end{array}$$

Hence, x^2+2x-3 is the H. C. F. required. See *Handbook*, page 105.

4. (a) Product of 1st and 2nd factors $= x^2 + a^2$ (A.)

$$\text{" " 3rd and 5th " } = x^2 - \frac{a^2}{2}(1 + \sqrt{-3}) \quad (\text{B.})$$

$$\text{" " 4th and 6th " } = x^2 - \frac{a^2}{2}(1 - \sqrt{-3}) \quad (\text{C.})$$

Product of B and C = product of

$$\left(x^2 - \frac{a^2}{2}\right) - \sqrt{-3} \text{ and } \left(x^2 - \frac{a^2}{2}\right) + \sqrt{-3}$$

$$= x^4 + \frac{a^4}{4} - a^2x^2 + 3$$

$$= \frac{1}{4}(4x^4 - 4a^2x^2 + a^4 + 12). \text{ Multiply this by A}$$

$$\text{and product} = x^6 + 3x^2 + 3a^2 + \frac{a^6}{4}$$

(b) Put $\sqrt{4+3\sqrt{-20}}=x+y$

$$\therefore \sqrt{4-3\sqrt{-20}}=x-y. \text{ See text-books.}$$

Multiplying $\sqrt{16+180}=x^2-y^2=14$. Again squaring I and equating rational parts, $x^2+y^2=4$. $\therefore x^2=9, x=\pm 3$.

But given expression $=(x+y)+(x-y)=2x=\pm 6$. Q.E.D.

5. (a) Squaring $x^4-14x^2+49=x^2-42x+89$. Add to both sides $8x^2-40$, and $x^4-6x^2+9=9x^2-42x+49$

$$\therefore x^2-3=\pm(3x-7)$$

$$\therefore x^2-3x+4=0, \text{ or } x^2+3x-10=0$$

$$\therefore x=\frac{1}{2}(4\pm\sqrt{-7}), \text{ or } x=2, -5.$$