10. $x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=m$; i.e. $\left(x+\frac{1}{x}\right)^3$

$x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)=n$; i.e. $\left(x-\frac{1}{x}\right)^3$

$\left.\begin{array}{l} =m; \therefore x+\frac{1}{x}=\sqrt[3]{m} \\ =n; \therefore x-\frac{1}{x}=\sqrt[3]{n} \end{array}\right\}$, adding and subtracting we obtain $x=\frac{\sqrt[3]{m}+\sqrt[3]{n}}{2}$ and $\frac{1}{x}=\frac{\sqrt[3]{m}-\sqrt[3]{n}}{2}$; $\therefore 4=m^{\frac{2}{3}}-n^{\frac{2}{3}}$

11. If $x^2+py^2+qz^2$ is $\div$ble by $x^2-(ay+bz)x+abyz$, it is $\div$ble by $(x-ay)(x-bz)$ $\therefore$ putting $x=ay$ we obtain $a^2y^2+py^2+qz^2=o,^{(1)}$.

Similarly putting $x=bz$ we obtain $b^2z^2+py^2+qz^2=o,^{(2)}$ $\therefore a^2y^2=b^2z^2$ and $z^2=\frac{a^2y^2}{b^2}$ substituting this value in (1) $a^2y^2+py^2+q\frac{a^2y^2}{z^2}=o$; divide through by a^2y^2, then $1+\frac{p^2}{a^2}+\frac{q}{b^2}=o$.

12. Since a, b, c are in H. P. $b=\frac{2ac}{a+c}$ or $2b^2=\frac{8a^2c^2}{(a+c)^2}$

Now $a^2+c^2>2ac$; $\therefore a^4+2a^2c^2+c^4>4a^2c^2$ (1), also $a^2+c^2>2ac$ and $2ac=2ac$; $\therefore 2ac(a^2+c^2)>4a^2c^2(2)$; $\therefore$ adding (1) and (2), $(a^2+c^2)^2+2ac(a^2+c^2)>8a^2c^2$

$\therefore a^2+c^2>\frac{8a^2c^2}{(a+c)^2}>2b^2$.

13. (a). Let $s=1^2x+2^2.x^2+3^2.x^3+4^2x^4$&c. then $s\,x=1^2.x^2+2^2.x^3+3^2.x^4+4^2.x^5$ &c.

$\therefore s(1-x)=1^1.x+3x^2+5x^3+7x^4+$&c.

$\therefore s\,x(1-x)=1^1.x^2+3x^3+5x^4+7x^5+$&c.

$\therefore s(1-x)^2=x+2x^2+2x^3+2x^4+$&c.

$\therefore s(1-x)^2=x+\frac{2x^2}{1-x}=\frac{x(1+x)}{1-x}$

$\therefore s=\frac{x(1+x)}{(1-x)^3}$.

(b). $\frac{1^2}{2}+\frac{2^2}{2^2}+\frac{3^2}{2^3}+$ &c. $=\frac{\frac{1}{2}(1+\frac{1}{2})}{(1-\frac{1}{2})^3}$

(since $x=\frac{1}{2}$) $\qquad =\frac{\frac{3}{4}}{\frac{1}{8}}=6$.

14. Let x = 1st term and d common diff. of corresponding Arith. Progression, then :—

$\frac{1}{x}-\frac{1}{x+d}=\frac{1}{4a+2b+c}$ (1),

$\frac{1}{x+d}-\frac{1}{x+2d}=\frac{1}{9a+3b+c}$ (2),

$\frac{1}{x+2d}-\frac{1}{1c+3d}=\frac{1}{16a+4b+c}$ (3);

$\therefore \frac{x(x+d)}{d}=4a+2b+c$ (4) or $x^2+dx=d(4a+2b+c)$ (7)

$\frac{(x+a)(x+2d)}{d}=9a+3b+c$ (5) or $x_2+3dx+2d^2=d(9a+3b+c)$ (8)

$\frac{(x+2d)\ (x+3d)}{d}=16a+4b+c$ (6) or $x_2+5dx+6d_2=d(16a+4b+c)$ (9)

from (7) and (8) $2dx+2d_2=d(5a+b)$ (10)

from (8) and (9) $2dx+4d^2=d(7a+b)$ (11)

$\therefore 2d^2=d(2a)$ or $d=a$,

Substituting a for d in (10), we obtain $x=\frac{3a+b}{2}$

$\therefore$ substituting a for d and x in (4) and simplifying we obtain $b_2=a_2+4ac$.

15. The sum of the products will be the coefficient of x^m in the product of $(1+cx)(1+^2cx)(1+c^3x)\ldots\ldots(1+c^nx)$.

Let $(1+cx)(1+c^2x)(1+c^3x)\ldots\ldots(1+c^nx)=1+A_1x+A^2x+\ldots\ldots A_nx^n\ldots$ write for x, cx, then

$(1+c^2x)(1+c^3x)(1+c^4x)\ldots\ldots(1+c^nx)(1+c^{n+1}x)=1+A_1cx+A_2c^2x+\ldots A_nc^nx^n$

i.e. $\frac{\left(1+A_1x+A_2x_2..A_nx^n\right)\left(1+c^{n+1}x\right)}{1+cx}$

$=1+A_1cx+A_2c^2x^2+\ldots A_nc^nx^n$.

Multiply out and equate coefficients ; then

$A_r+A_{r-1}c^{n+1}=A_rc^r+A_{r-1}c^r$;

$\therefore A_r=\frac{A_{r+1}\left(c^{n+1}-c\right)}{c-1}$. Giving r values, 1, 2, 3,.... we get