GRAPHICAL TREATMENT OF ELASTIC RIBS.

I. BEAMS.

THE deflection of curved or straight beams or arch ribs can easily be determined analytically, according to C. S. Whitney, M.C.E., in "The Cornell Civil Engineer," by the use of the calculus for very simple cases, but where the shape of the rib is irregular or the moment of inertia is variable so that it cannot be expressed by a simple equation the treatment becomes

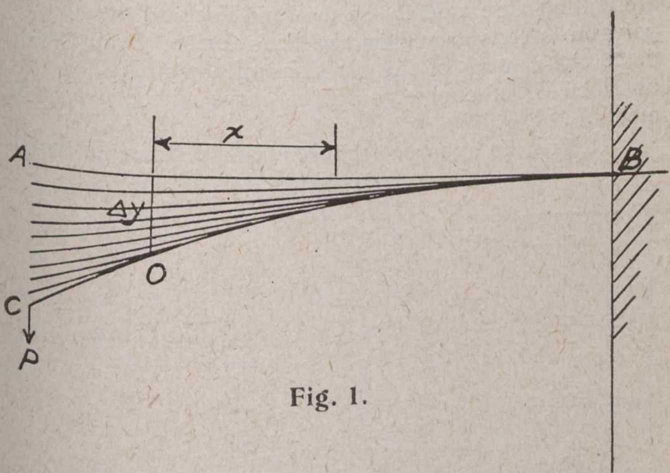

Fig. 1.

quite difficult. By the remarkably simple method explained below it is possible to draw to scale the elastic curve for any rib or beam under any load in a very short time. The method seems to be practically unknown in this country, and although it is original with the writer

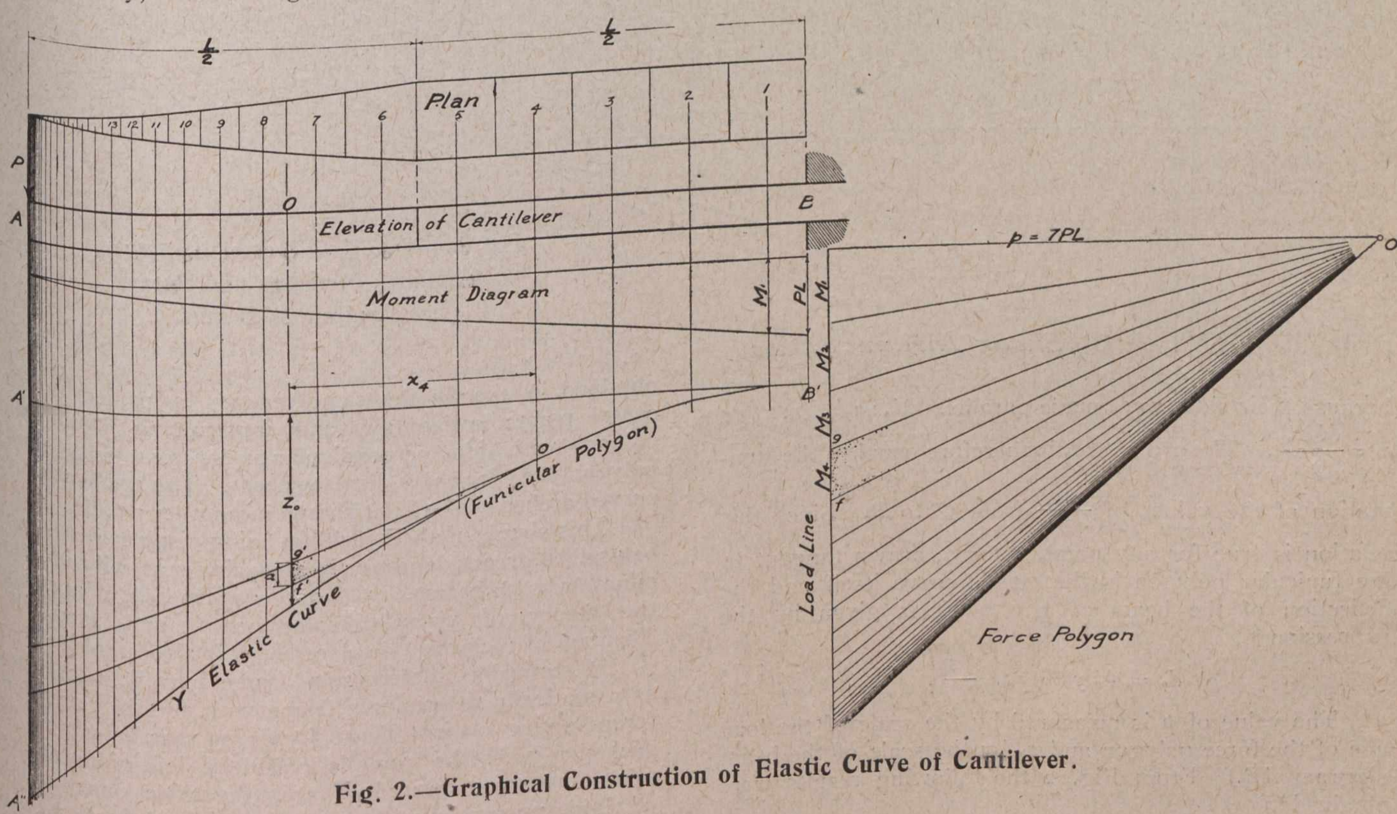

Fig. 2.—Graphical Construction of Elastic Curve of Cantilever.

he has found that it has been in use in Europe for some time.

Although American engineers prefer the theory of least work in solving statically indeterminate structures, the theory of virtual displacements is often much simpler; and, aside from its use in determining deflections, the method here outlined in combination with the theory of virtual displacements affords the simplest of all methods of analyzing such structures as continuous beams, two-hinged arches, or fixed reinforced concrete arches. Asymmetry or irregularity of section does not affect the ease of solution. Results can be obtained by the graphical method with any degree of accuracy which may be warranted by practical considerations.

The deflections are obtained by the graphical integration of the familiar equations

$$\Delta x = \int \frac{M\,y\,ds}{EI},\quad \Delta y = \int \frac{M\,x\,ds}{EI},\ \text{and}\ \Delta \phi = \int \frac{M\,ds}{EI},$$

the derivation of which may be found in Church's "Mechanics of Engineering" and other texts. It must be noted that the integration is from the point of which the displacements are to be obtained to the point where the rib is considered fixed and the displacements secured are relative to the tangent and normal at the point considered fixed.

In the case of the simple straight cantilever (Fig. 1) the y displacement, Δy, of any point O relative to the tangent AB is equal to $\int_0^B \frac{M\,x\,ds}{EI}$ or if the beam be divided into small Δ s's the value of Δy is approximately $\sum_0^B \frac{M\,x\,\Delta s}{EI}$. If the value of $\frac{\Delta s}{EI}$ be made a constant, then

$$\Delta y = \frac{\Delta s}{EI} \sum_0^B M x \qquad \text{Eq. 1.}$$

The simplicity of this equation suggests a graphical summation which may be accomplished by means of the force polygon and funicular polygon, as shown in Fig. 2.

As an example, we will consider a cantilever beam with a wedge-shaped end. The beam is first divided into small lengths so that $\frac{\Delta s}{EI}$ is a constant. A moment diagram is then drawn for the assumed condition of loading which will be taken as a concentrated load at the end.