To shew that $(x^2+3)(x^2+7)$ is divisible by 32, let x be any *odd* number greater than unity; then $(x^2+3) = (x+1)(x-1)+4$. Now x is odd; therefore $(x+1)$ and $(x-1)$ are each divisible by 2; therefore the quantity $(x+1)(x-1)+4$ or x^2+3 is divisible by 4. So $(x^2+7)=(x+1)(x-1)+8$. Again, each of the factors $(x+1)$ and $(x-1)$ is divisible by 2, and one of these quotients is again divisible by 2; therefore the quantity $(x+1)(x-1)+8$, or x^2+7 can be divided by 8 and the whole expression $(x^2+3)(x^2+7)$ is divisible by 8×4, or 32.

128. The sum of the squares of three consecutive numbers, when increased by unity, is divisible by 12 but never by 24.

Let x be any even number, then $x-1$, $x+1$ and $x+3$ are three consecutive odd numbers. Square each, add together and increase the sum by unity; the result is $3x^2+6x+12$, which is $3(x^2+2x+4)$. Now x is *even*; therefore x^2 is divisible by 4, as is also $2x$; therefore 4 is a factor of x^2+2x+4, and $3(x^2+2x+4)$ is divisible by 3×4, or 12.

129. If $x=(p+q)^2$, find value of
$$(p^2+q^2)x-2pqx-(q^4+p^4)$$

Substitute value of x in the given expression:

$$(p^2+q^2)(p+q)^2-2pq(p+q)^2-(q^4+p^4)$$
$$=(p^2+q^2)^2+2pq(p^2+q^2)-2pq(p^2+q^2)-4p^2q^2-(q^4+p^4)$$
$$=2p^2q^2-4p^2q^2=-2p^2q^2$$

130. Shew that

$$(x-y)^3+(y-z)^3+(z-x)^3 =3(x-y)^2+3(y-z)^2+3(z-x)^2-6(x-y)(y-z)(z-x).$$

$$(x-y)+(y-z)+(z-x)=0$$
$$\{(x-y)+(y-z)+(z-x)\}^3=0$$

$(x-y)^3+(y-z)^3+(z-x)^3=-3(x-y)^2\{(y-z)+(z-x)\}-3(y-z)^2\{(x-y)+(z-x)\}-3(z-x)^2\{(x-y)+(y-z)\}-6(x-y)(y-z)(z-x)$.

$(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)^2(x-y)+3(y-z)^2(y-z)+3(z-x)^2(z-x)-6(x-y)(y-z)(z-x)$; that is, $(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)^3+3(y-z)^3+3(z-x)^3-6(x-y)(y-z)(z-x)$.

131. Shew that
$$abc>(a+b-c)(a+c-b)(b+c-a).$$

If one relation exists between the values of a, b and c, this proposition is not true; point out that relation.

$$\left\{\begin{array}{l}1.\ a^2>a^2-(b-c)^2;\ \therefore a^2>(a-b+c)(a+b-c)\\ 2.\ b^2>b^2-(a-c)^2;\ \therefore b^2>(b-a+c)(b+a-c)\\ 3.\ c^2>c^2-(a-b)^2;\ \therefore c^2>(c-a+b)(c+a-b)\\ \quad\text{Multiplying, we get}\\ 4.\ a^2b^2c^2>(a-b+c)^2(b-a+c)^2(a+b-c)^2\ \therefore\\ 5.\ abc>(a-b+c)(b-a+c)(a+b-c).\end{array}\right.$$

132. Two spheres, whose radii are x and y, touch each other internally. Find the distance of the centre of gravity of the solid contained between the two surfaces, from the point of contact.

Let m_1 = volume of larger sphere whose radius is x, and m_2 = volume of smaller sphere whose radius is y. The centre of gravity of a sphere is at its centre. Let v = the distance of the centre of gravity of the solid contained between the surfaces from the point of contact. Then

$$v(m_1-m_2)=m_1x-m_2y$$
$$v=\frac{m_1x-m^2y}{m_1-m_2},$$

but $m_1=\frac{4}{3}\pi x^3$ and $m_2=\frac{4}{3}\pi y^3$.

Substituting for m_1 and m_2 we get

$$v=\frac{\frac{4}{3}\pi x^4-\frac{4}{3}\pi y^4}{\frac{4}{3}\pi x^3-\frac{4}{3}\pi y^3}=\frac{x^4-y^4}{x^3-y^3}=\frac{(x+y)(x^2+y^2)}{x^2+xy+y^2}.$$

W. J. ELLIS, B.A.,
Math. Master, Coll. Institute, Cobourg.

137. The sides of a triangle ABC are divided in order in ratio $m:n$, and the points of section joined and a new triangle formed; the sides of this being divided as before, and so on to the r^{th} triangle.

Shew that the areas of the given triangle and the r^{th} inscribed triangle are in ratio $\left\{\frac{(m+n)^2}{m^2-mn+n^2}\right\}^r$, and also that any rational algebraic function of the areas of the odd triangles of the series will be to the same function of the areas of the even triangles in ratio $\frac{(m+n)^2}{m^2-mn+n^2}$.