by the subscripts e, ev, etc., with multiples of n rejected. Consequently, in determining $R_e^{\frac{1}{n}}$, $R_{ev}^{\frac{1}{n}}$, etc., as in (76), (77), etc., we have determined all the terms

$$R_1^{\frac{1}{n}},\ R_2^{\frac{1}{n}},\ \ldots\ldots,\ R_{n-1}^{\frac{1}{n}}. \tag{78}$$

Substitute, then, in (73) the rational value which $R_0^{\frac{1}{n}}$ can be shown, as in § 8, to possess, and the values of the terms in (78) as these are determined in (76), (77), etc., and the root is constructed; in other words, the expression (73) shall be the root of a pure uni-serial Abelian equation of the n^{th} degree, provided always that the equation of the n^{th} degree, of which it is the root, is irreducible.

Necessity of the Above Forms.

§ 42. Here we assume that the root of a pure uni-serial Abelian equation $f(x) = 0$ of the n^{th} degree is expressible as in (73), and we have to prove that its fundamental element R_1 has the form (72), and that the terms in (78) are to be taken as in (76), (77), etc., while $R_0^{\frac{1}{n}}$ receives its rational value.

§ 43. By (3), z being any integer,

$$R_{z\sigma}^{\frac{1}{n}} = \{F(w)\}\, R_1^{\frac{z\sigma}{n}},$$

$F(w)$ being a rational function of w. And equation (5) subsists along with (3); that is, w^e being the general primitive n^{th} root of unity,

$$R_{ez\sigma}^{\frac{1}{n}} = \{F(w^e)\}\, R_e^{\frac{z\sigma}{n}}.$$

Taking $z = 1$,

$$R_{e\sigma}^{\frac{\lambda^{s-2}}{n}} = B_e R_e^{\frac{\sigma\lambda^{s-2}}{n}},$$

B_e being a rational function of w^e. In like manner, taking $z = \lambda$,

$$R_{e\sigma\lambda}^{\frac{\lambda^{s-3}}{n}} = C_e R_e^{\frac{\sigma\lambda^{s-2}}{n}},$$

C_e being a rational function of w^e. In this way it can be shown that each of the terms in the series

$$R_{e\sigma}^{\frac{\lambda^{s-2}}{n}},\ R_{e\sigma\lambda}^{\frac{\lambda^{s-3}}{n}},\ R_{e\sigma\lambda^2}^{\frac{\lambda^{s-4}}{n}},\ \ldots\ldots,\ R_{e\sigma\lambda^{s-2}}^{\frac{1}{n}},$$

is the product of $R_e^{\frac{\sigma\lambda^{s-2}}{n}}$ by a rational function of w^e. Therefore

$$(R_{e\sigma}^{\lambda^{s-2}} R_{e\sigma\lambda}^{\lambda^{s-3}} R_{e\sigma\lambda^2}^{\lambda^{s-4}} \ldots\ldots R_{e\sigma\lambda^{s-2}})^{\frac{\sigma}{n}} = F_e R_e^{\frac{d}{n}} \tag{79}$$

where F_e is a rational function of w^e, and

$$d = \sigma^2 (s-1)\, \lambda^{s-2}. \tag{80}$$

So also

$$\left.\begin{array}{l} (R_{e\tau}^{h^{t-2}} R_{e\tau h}^{h^{t-3}} \ldots\ldots R_{e\tau h^{t-2}})^{\frac{\tau}{n}} = G_e R_e^{\frac{\delta}{n}} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ (R_{e\beta}^{k^{b-2}} R_{e\beta k}^{k^{b-3}} \ldots\ldots R_{e\beta k^{b-2}})^{\frac{\beta}{n}} = H_e R_e^{\frac{D}{n}} \end{array}\right\} \tag{81}$$