

By (5) and (6)

$$A(g^2 - y) = g(k^2 - t^2y) + azA',$$

and

$$p_4 = -20A + 5g^2 + 15y.$$

Therefore

$$azA' = \frac{g^2 - y}{20} (5g^2 + 15y - p_4) - g(k^2 - t^2y). \quad (35)$$

§19. From the second of equations (6),

$$p_5(g^2 - y)^2 + 4B(g^2 - y)^2 - 40ty(g^2 - y)^2 = 0.$$

Therefore, from the first of the two equations (34),

$$(p_5 - 40ty)(g^2 - y)^2 + 4(g^2 + y)P + 8gyQ + 8azA'\{t(g^2 + y) + 2gk\} = 0.$$

Putting for azA' its value in (35),

$$\begin{aligned} (g^2 - y) \left[(p_5 - 40ty)(g^2 - y) + 2 \left(g^2 + 3y - \frac{1}{5}p_4 \right) \{t(g^2 + y) + 2gk\} \right] \\ + [4(g^2 + y)P + 8gazQ - 8g(k^2 - t^2y)\{t(g^2 + y) + 2gk\}] = 0. \end{aligned} \quad (36)$$

But, from the values of P and Q ,

$$\begin{aligned} 4(g^2 + y)P + 8gazQ - 8g(k^2 - t^2y)\{t(g^2 + y) + 2gk\} \\ = \{ -4(g^2 + y)(gk - ty) + 8gy(k - gt) - 8(k + gt)(k^2 - t^2y)\}(g^2 - y). \end{aligned}$$

Therefore, rejecting the common factor $g^2 - y$, (36) becomes

$$\begin{aligned} (p_5 - 40ty)(g^2 - y) + 2 \left(g^2 + 3y - \frac{1}{5}p_4 \right) \{t(g^2 + y) + 2gk\} \\ - 4(g^2 + y)(gk - ty) + 8gy(k - gt) - 8(k + gt)(k^2 - t^2y) = 0. \end{aligned}$$

Arranging according to the powers of y ,

$$\left. \begin{aligned} 50gt^2t + y \left\{ 8gt^2 + 8kt^2 - t \left(20g^2 + \frac{2}{5}p_4 \right) - p_5 \right\} \\ + t \left\{ 2g^3 \left(g^2 - \frac{p_4}{5} \right) - 8gk^2 \right\} + g^2p_5 - 8k^3 - \frac{4}{5}gkp_4 = 0. \end{aligned} \right\} \quad (37)$$

This is one equation between the unknown quantities y and t .

§20. From the last two of the equations (6),

$$he^2(\theta^2 + \phi^2z) = (k^2 + t^2y) - 2kat + (g^2 - y)(a - g)$$

and

$$2kze^2(\theta\phi) = 2kzat - (k^2 + y^2t) - (g^2 - y)(az - g).$$

But

$$(\theta^2 - \phi^2z)^2 = (\theta^2 + \phi^2z)^2 - 4z\theta^2\phi^2.$$

Therefore

$$\begin{aligned} zhe^2e^4(\theta^2 - \phi^2z)^2 = z \{ (k^2 + t^2y) - 2kat + (g^2 - y)(a - g)^2 \} \\ - \{ 2kzat - (k^2 + t^2y) - (g^2 - y)(az - g)^2 \}. \end{aligned}$$