which again reduces to $1-(1-\sin^2A + \sin^2B)$ or, $\sin^2A+\sin^2B$.

$\therefore$ The left-hand fraction becomes $\dfrac{\sin^2A+\sin^2B}{\sin^2A+\sin^2C}$, which $=\dfrac{a^2+b^2}{a^2+c^2}$ by 5(a).

6. (a) Find an expression for the area of an n-sided regular polygon.

Let s be a side of the polygon, θ be the angle at the centre subtended by a side, and p be the apothem. Then $p={}^s/_2\cot{}^\theta/_2$, and the area of one of the triangles included by two radii and a side is $p.{}^s/_2$.

$$\therefore \text{The whole area} = n\frac{s^2}{4}\cot\frac{\theta}{2} = n\frac{s^2}{4}\cot\frac{\pi}{n}$$

The area might also be found in terms of the perimeter, or of the circum-radius, etc.

(b). Show that $2R+2r=a\cot A+b\cot B+c\cot C$.

Remembering that ${}^a/_{2R}=\sin A$, etc., we divide through by $2R$ and obtain : $\cos A+\cos B+\cos C=1+{}^r/_R$

But $\cos A+\cos B+\cos C=1+4\sin{}^A/_2 . \sin{}^B/_2 . \sin{}^C/_2$ by a known theorem; and $\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$, etc. And this expression becomes,

$$1+4\sqrt{\left\{\frac{(s-b)(s-c)}{bc} . \frac{(s-c)(s-a)}{ca} . \frac{(s-a)(s-b)}{ab}\right\}}$$

$$=1+\frac{4\Delta^2}{sabc}=1+\frac{4\Delta}{abc} . \frac{\Delta}{s}=1+\frac{r}{R}. \quad \text{q. e. d.}$$

7. From a station, B, at the base of a mountain, its summit, A, is seen at an elevation of 60^0; after walking one mile towards the summit up an incline of 30^0 with the horizon to another station, C, the observer finds the angle BCA to be 135^0; find the height of the mountain.

Evidently the elevation of the summit as seen from station C is 75^0.

Let h be the height of the mountain, and x be the distance from B to the foot of the perpendicular let fall from the summit.

Then h tan 60^0; and going to station C, $h-\sin 30^0=(x-\cos 30^0)$ $\tan 75^0$. And substituting for x, $h-\sin 30^0=(h\cot 60^0-\cos 30^0)\tan 75^0$;

$$\text{whence } h=\frac{\sin 30^0-\cos 30^0 . \tan 75^0}{1-\cot 60^0 . \tan 75^0}$$

Otherwise as follows : $\angle BAC=180^0-(135^0+30^0)=15^0$.

Then $BA : BC=\sin 135^0 : \sin 15^0$

$\therefore BA=BC . \dfrac{\sin 45^0}{\sin 15^0}$. And the height of the mountain is $BA \sin 60^0=$

$$BC . \frac{\sin 45^0 . \sin 60^0}{\sin 15^0}$$

Both of these solutions give the same result upon substituting the values of the functions, namely, the height $=\frac{1}{2}(3+\sqrt{3})$, or 2.366 + miles.

8. (a) Prove that $\log_a \sqrt[k]{m}={}^1/_k\log_a m$, and that $\log_b N=\dfrac{\log_a N}{\log_a b}$.

1st. Let $a^p=m$. Then by definition $p=\log_a m$

And $a^{p/k}=m^{1/k}=\sqrt[k]{m}$. $\therefore {}^p/_k=\log_a\sqrt[k]{m}$. Whence ${}^1/_k\log_a m=\log_a\sqrt[k]{m}$.

2nd. Let $b^x=N$. Then $x=\log_b N$. Also take logarithms to base a.