$$\frac{(xy+1+2x)(xy+1+2y)+(x-y)^2}{x^2y^2+1-x^2-y^2}$$

7. What is meant by the root of an equation? Shew how to find those of a quadratic, and investigate the relations between them and the coefficients of the equation.

Find the conditions that the equations

$$ax^2+bx+c=0$$
$$px^2+qx+r=0$$

may have (1), one root common; (2), roots the same in magnitude, but of contrary signs.

8. Solve the equations:

(1). $\frac{x+1}{2}-\frac{2x-1}{3}=\frac{3x+4}{4}-\frac{5x-6}{3}$;

(2). $\frac{(x-1)(x+4)}{x+3}=\frac{(3+x)(2-x)}{1-x}$

(3). $\frac{1+2x}{1-2x}=\frac{1+x+\sqrt{1+2x}}{1-x-\sqrt{1-2x}}$

9. Find x from the equation

$$\frac{(x-1)^2\ (n-1)^2+4n}{(x+1)^2\ (n-1)^2+4n}=p$$

and shew that if n be positive and x real the value of the left hand member always lies between n and $\frac{1}{n}$.

10. Find the Arithmetic, Geometric and Harmonic means between two given quantities; for example, between $\frac{3}{4}$ and $\frac{4}{3}$.

If H be the Harmonic mean between a and b it is also the Harmonic mean between $H-a$ and $H-b$.

11. Investigate the formulas for the sum of a series of quantities in Arithmetic or Geometric progession.

(1). Find the 37th term of $6+\frac{35}{6}+\frac{17}{3}+\ldots$ and the sums of 31 and 42 terms.

(2). Find the sum of n terms of

$$3\tfrac{1}{5}+2+1\tfrac{1}{4}+\ldots$$

(3). Of $1-0.4+0.16-0.064+\ldots$ find the sum to infinity, and find the least number of terms of which the sum will agree with this as far as 8 places of decimals.

12. There are p arithmetic series each continued to n terms; their first terms are the natural numbers 1, 2, 3 ... , and their common differences are the successive odd numbers 1, 3, 5 ....; prove that the sum of all of them is the same as if there were n such series each continued to p terms.

SOLUTIONS TO PROBLEMS FROM CORRESPONDENTS.

1. To produce a given straight line so that the rectangle contained by the whole line thus produced and another given straight line may be equal to the square on the produced part.

Let a, b denote the lengths of the two given lines and x that of the line required.

Then since $(a+x)b=x^2$

$$\therefore\ (x-\tfrac{1}{2}b)^2=\frac{4ab+b^2}{4}$$

$$\therefore\ x=\tfrac{1}{2}b+\tfrac{1}{2}\sqrt{4ab+b^2}$$

Hence we obtain the geometrical construction: To four times the rectangle contained by the two lines a and b add the square on b; construct a square equal to this resulting rectangle; to half the side of this square add the half of b and the line required is obtained.

2. The difference of the angles at the base of a triangle is double the angle contained by the line bisecting the vertical angle, and the line drawn from the vertical angle perpendicular to the base.

Let ABC be the triangle. Draw CD perpr. to AB and CE, bisecting the angle C. Let angle A be greater than B, then the point E lies between D and B.

$\therefore$ ACD + CAD = 2 rt. angles
= BCD + CDB
$\therefore$ A − B = BCD − ACD
But BCD = $\frac{1}{2}$C + DCE
and ACD = $\frac{1}{2}$C − DCE
$\therefore$ BCD − ACD = 2 DCE
$\therefore$ A − B = 2 DCE.

3. The price of diamonds per carat varies as their weight. If a diamond of 3 carats be worth \$342, find the value of a diamond of 4 carats.

If a diamond of 3 carats is worth \$342 it is worth \$114 per carat, $\therefore$ a diamond of 4 carats