❋ Mathematics. ❋

All communications intended for this department should be sent before the 20th of each month to Chas. Clarkson, B.A., Seaforth, Ont.

INVERTING THE DIVISOR.

On page 316 of The Journal, March 1st, there is an extract from the *Ohio Educational Monthly*, in which the following sentences occur. The writer, Mr. Lloyd Wyman, says: "My experience shows me that the great majority of pupils...... even of the High Schools, are unable to explain intelligently the processes of multiplying and dividing fractions......Is this a reflection on the teachers? I think so......In division of fractions, why not divide each term of the dividend by the corresponding term of the divisor?......Take, for example, the following question: Required, the quotient of $\frac{15}{32} \div \frac{5}{8}$. Dividing each term of the dividend by the corresponding term of the divisor, the correct quotient $\frac{3}{4}$ is immediately found, and the reasoning is plainly apparent, viz., dividing by 15 by 5 brings a quotient 8 times too small; therefore *increase the size of the denominator* 8 times *by dividing* 32 by 8." [Note the confusion of thought in the words we have italicized.]......"Even when the terms are not thus exactly divisible, the process can be indicated; thus, $\frac{3}{4} \div \frac{2}{3} = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{9}{8} = 1\frac{1}{8}$, and by so doing we guard against propagating that unknowable mystery of 'inverting the divisor.'"

This may pass muster in Ohio, but it will not do for the Province of Ontario. It is a good example of the logical fallacy of stating the same thing under two different names, and then giving one as the reason of the other. The quotient of $\frac{3}{4} \div \frac{2}{3}$ is re-stated in the form of a complex fraction where the line dividing the numerator from the denominator is precisely equivalent to the sign $\div$, and the "unknowable mystery" is solved by substituting one sign of operation for another sign of operation. Is the second mystery any more fathomable than the first? Is it easier and more lucid to say, "Multiply the extremes for a new numerator, etc." than to say, "Invert the divisor, etc.?" The *verve* and cocksure confidence of the writer remind us of what one sometimes hears at Teachers' Institutes even in Ontario, when some callow member of our honorable profession rises to expatiate on "My Method of Teaching Fractions." Mr. Wyman has evidently supposed that his "method" will quickly remove "this reflection on the teachers;" but it is quite certain that he has never tried the experiment on actual living, laughing pupils, or he would be suddenly undeceived. He would find that the second mystery which he substitutes for the first is not one whit less impenetrable to the intellect. Whoever can explain clearly the process of reducing the complex fraction, can also explain the reason for inverting the divisor, and Mr. W. has only darkened counsel by a multitude of words. Heaven help the pupil who has to learn arithmetic from a text-book written by Mr. Lloyd Wyman! He would be in a worse dilemma than the Ontario pupil who has to work out his arithmetical salvation by digesting the "Public School Arithmetic" authorized by our sapient Education Department to the exclusion of a really good book which we formerly used. The greatest wonder is that the teachers of this Province will tamely submit to such a caricature without protesting vigorously and persistently until a better book is supplied, the result achieved in the case of the unutterably bad "Public School History," which the Minister assured us had come to stay.

But to return to these fractions. The first point to be recognized is the fact that the division of whole numbers and the division of fractions are only two examples of the same process. There are not two kinds of division any more than there are two sorts of addition. The difficulty with fractions can only be reduced by a short process of inductive reasoning. If a boy is asked to divide 7 yards by 4 inches he knows at once that the quantities must be expressed in the same denomination as the first step. Very well, if 3 *fourths* are to be divided by 2 *thirds*, must not the same principle hold good? Must he not first reduce them both to *twelfths*, and say 9 *twelfths* divided by 8 twelfths? The answer is, of course, $9 \div 8$; and the teacher then points out the mechanical rule which will give the same result in all cases, without the trouble of reasoning out the result for each particular example. He shows that $\frac{9}{8} = \frac{3 \times 3}{4 \times 2} = \frac{3}{4} \times \frac{3}{2}$. "*A rule is of no use until it is of no use*," *i.e.*, until the application has become a matter of unconscious cerebration. When a pupil has gone through the reasoning process a dozen times and has become thoroughly convinced of the essential truth of the result, he may properly take the shortest cut in his work to arrive at the result which he *knows* to be correct. It is absurd to suppose that he must continuously repeat the PROOF of a result, and not avail himself of the quickest means of finding what he already *knows* to be correct. See what would happen in the practical world of morals, of business, of politics, if mechanical rules were not learned once for all as the result of careful reasoning! In order to *confirm the faith* of the pupil the teacher ought to demonstrate the rule in as many ways as possible, and compel the pupil to repeat the reasoning *with a different example*, always choosing the simplest plan first.

In the case under consideration, he may at the second lesson substitute another method of arriving at the rule, and say $\frac{3}{4} \div \frac{2}{5}$ = the quotient, whatever it may be.

$\therefore 3 \div \frac{2}{5} = 4$ times the quotient; for, 3 is four times $\frac{3}{4}$.

$\therefore 3 \div 2 = \frac{4}{5}$ times the quotient; for, 2 is five times greater than $\frac{2}{5}$, and when the divisor is five times greater the quotient must be five times smaller than before;

$\therefore \frac{3}{2} = \frac{4}{5}$ of the quotient. Multiply these equals by $\frac{5}{4}$

and $\frac{3}{2} \times \frac{5}{4} = \frac{4}{5} \times \frac{5}{4}$ quotient.

$\therefore \frac{3 \times 5}{2 \times 4}$ = quotient $= \frac{3}{4} \times \frac{5}{2}$, which is the rule.

At the third lesson he may say $\frac{3}{4} \div \frac{2}{5}$ means $\frac{3}{4} \div$ *one-fifth* of 2, and the result must be *five* times greater than $\frac{3}{4} \div$ the whole of 2.

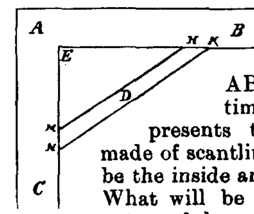
But $\frac{3}{4} \div 2$ is $= \frac{3}{8}$, *for the* SIZE *of eighths is only* half the size of fourths. Hence $\frac{3}{4} \div \frac{2}{5}$ must be FIVE times $\frac{3}{8}$, which is $\frac{15}{8}$. And this is $\frac{3 \times 5}{2 \times 4}$, and we see that this is the same as $\frac{3}{2} \times \frac{5}{4}$, from the rule for multiplication learned in previous lessons.

None of the rules of arithmetic should be learned before they have been properly established by reasoning, and all this reasoning must be made to depend on the fact that a fraction expresses the NUMBER of *equal* parts by means of the *numerator*, and the SIZE of those *equal* parts by means of the *denominator*. If we increase the NUMBER of parts, by adding something to the numerator, or by multiplying the numerator, we increase the value of the fraction in the same proportion. If we decrease the NUMBER of equal parts by subtracting something from the numerator, or by dividing the numerator, we *decrease* the value of the whole fraction. The reverse is true for the denominator. If both processes take place together in the same proportion, the value of the fraction remains the same. If these principles, few and simple, are properly understood by reference to ordinary division of whole numbers, there is no "unknowable mystery" at all; the pupil proceeds in the clear sunshine of undoubted certainty, and the mechanical rules are to him only short processes of attaining results of which he is already *sure*, just as Homer's method of extracting the cube root, later on, is only an ingenious device for shortening the labor of applying $(a+b)^3$ to numbers.

In conclusion, we would like to ask the teachers of Canada whether they are satisfied with a book like the authorized "High School Arithmetic," which eliminates the theory of arithmetic and substitutes for it a good collection of problems. How are the teachers of the future to know the *science* of arithmetic, if they have to depend entirely on the oral explanations of their own teachers? Will they pay any attention to a subject that is practically tabooed on all our test examinations? Will they get a proper grasp of the theory at the Model Schools, or at the Normal Schools? Can they teach elementary classes efficiently without a comprehensive knowledge of the theoretical principles of arithmetic? Or, will they not be likely to substitute confused explanations, the fallacy of the circle, and *non sequiturs* for simple and sound reasons, as Mr. Lloyd has, with the purest motives, done in the article just quoted?

CORRESPONDENCE.

52. T.L.N. wishes a solution of the following question:—



The accompanying figure represents a brace which a carpenter is putting into a building. AB and AC represent the timber of the frame. D represents the brace. The brace is made of scantling 4 in. square. What will be the inside and outside length of brace? What will be distance from E to the centre of brace? What will be distance from M to N and from H to K?

The distance from E to H is 3 ft. and from E to M is 2 ft.

The carpenter wants to know what these distances would be if measured by the standard square.

J. T. White, A.M., Principal of Alleghany Co. High School No. 1, Cumberland, Maryland, sends the following problems:—

53. At what time after 10 o'clock is the hour-hand equally distant from the minute hand and from 12?

54. In a given square inscribe an equilateral triangle. Draw the figure and derive a general formula for similar problems.

55. Given $x = \frac{64\sqrt{x} - 105}{x - 42}$ to find x. Four values to be found.

56. Having a line equal to the sum of a side and the altitude of an equilateral triangle, to construct the equilateral triangle.

57. Having a line equal to the difference of a side and the altitude of an equilateral triangle, to construct the equilateral triangle.

58. Having a line equal to the sum of the three sides and the altitude of an equilateral triangle, to construct the equilateral triangle.

J.W.D., Waverly, asks for the solutions of the following questions taken from H. Smith's Arithmetic and Glashan's High School Arithmetic, page 274, No. 145, and page 261, No. 8, respectively:—

59. Two trains start at the same time, one from London to Norwich, the other from Norwich to London. They meet, after which they reach London and Norwich respectively 4 and 1 hrs. Show that one goes twice as fast as the other.

Also a purely arithmetical solution (if possible) of:

60. A father leaves $15,000 to be divided among his three sons, aged respectively 16, 18 and 20, so that if their respective shares be placed at simple interest at 6%, they may have equal shares on coming of age. How much does each now get?

61. A.B.C. asks the following question:—

"Could you give me the name of a work on algebra, one good on the general work and theory, as a companion to the present 'High School Algebra' and McLellan's work, to be used in private study?"

Answer.—See page 220, December, 1892, for reply to the same question, in which "Hall and Knight's" larger book and "Dupuis' Principles" are recommended, together with assistance by mail from some competent person. If A.B.C. will state his case in a private letter to the Mathematical Editor, he will receive a fuller answer.

SOLUTIONS.

62. K.S. wishes for explanation of the formula— $\log_a b \times \log_b a = 1$. He says he cannot understand the proof given in his text-book. We are not sure that we can make it less incomprehensible; however, we will try to help our friend.

Take any number $N = a^x = b^y$, then we have

$a = b^{\frac{y}{x}}$, or $\log_b a = \frac{y}{x}$;