MATHEMATICS.

Solutions to Algebra Paper in last number.

1. See Gross' Algebra, Chap. I.

2. Since any number and the sum of its digits when divided by 9 leave the same remainder, therefore, the number and the number reversed leave the same remainder on being divided by 9, we may therefore suppose

the number $= 9a + r$

and the number reversed $= 9b + r$

$\therefore$ 4 times the No. $= 36a + 4r$

5 times the No. reversed $= 45b + 5r$

the sum of these $= 36a + 45b + 9r$

which is exactly divisible by 9.

This will also be true if the second number instead of being the first number reversed be composed of the same digits in any order whatever; and also if the multipliers instead of being 4 and 5 be any two numbers whose sum is 9.

The same result may readily be shown to hold *mutatis mutandis* for any radix.

3. (1): $b^2 - 1$.

(2). $\left(\frac{xy}{3}\right)^2 - 3\left(\frac{xy}{3}\right) + 3$ by $\frac{xy}{3} + 3$

$$= \left(\frac{xy}{3}\right)^3 + (3)^3$$

(3). $a^2 + b^2 + c^2 + d^2$.

(4). The expression becomes

$$a^2(x+2)^2 - 2a(x+2) + 1$$

the square root of which is

$$a(x+2) - 1.$$

(5). $a + b - c$.

4. (1).

$$\begin{array}{r|rrrrr} 1 & 4 & +0 & +5 & +0 & +1 \\ 0 & & 0 & 0 & & \\ -2 & & & -8 & 0 & \\ 1 & & & & 4 & 0 \\ \hline & 4 & +0, & -3 & +4 & +1 \end{array}$$

$\therefore$ rem. is $-3x^2+4x+1$.

$$\begin{array}{r|rrrr} 1 & -3 & +4 & +1 & \\ 0 & & 0 & 0 & \\ -2 & & & +6 & -8 \\ 1 & & & & -3 \\ \hline & -3 & +4 & +7 & -11 \end{array}$$

which is $-\frac{3}{x} + \frac{4}{x^2} + \frac{7}{x^3} - \frac{11}{x^4}$

(2)

$$\begin{array}{r|rrrrrrr} & 1 & & & & & & \\ 1 & & 1 & +1 & 0 & -1 & -1 & \\ -1 & & & -1 & -1 & 0 & +1 & +1 \\ \hline & 1 & +1 & +0 & -1 & -1 & +0 & +1 \end{array}$$

$\therefore$ the series is $x + x^2 - x^3 - x^4 + x^6 +$ &c.

(3)

$$\begin{array}{r|rrr|r} 2 & 8 & +0 & -6 & +5 \\ -3 & & -12 & +18 & -18 \\ \hline & 4 & -6 & +6 & -13 \end{array}$$

$\therefore$ rem. is -13.

5. "Book work."

6. (1). $\frac{(a^2+b^2)^2}{(a^2-b^2)^2}$

(2). 1.

(3). $\frac{1}{(x-1)(x-2)(x-3)}$.

(4). Divide the denominators by

$$(x-1)(y-1)$$

then the fractions are equal if

$$(xy+1+2x)(xy+1+2y)+(x-y)^2$$
$$= (x+1)^2 (y+1)^2$$

which may readily be shewn by expanding both sides, or by factoring thus:

$$xy+1+2x)(xy+1+2y)$$
$$= (xy+x+x+1)(xy+1+2y)$$
$$= (xy+x+1)(xy+1+2y) + x(xy+1+2y)$$
$$= (xy+x+y+1)(xy+1+2y)$$
$$+ x(xy+1+2y) - y(xy+1+2y)$$
$$= (xy+x+y+1)(xy+1+2y)$$
$$+ (x-y)(xy+1+2y)$$

similarly $(xy+x+y+1)(xy+1+2y)$

$$= (xy+x+y+1)(xy+x+y+1)$$
$$-(x-y)(xy+x+y+1)$$

$$\therefore (xy+1+2x)(xy+1+2y) + (x-y)^2$$
$$= (xy+x+y+1)^2$$
$$= (x+1)^2(y+1)^2$$