ence is 3·1416 times the diameter; therefore, knowing the number of teeth and the pitch, we can easily find the diameters of the pitch circles. The number 3·1416 is usually denoted by the Greek letter π. Using decimals in our calculations we have $\frac{3}{4}'' = {\cdot}75''$; therefore the circumference of A $= {\cdot}75'' \times 24$; and the diameter $= \frac{{\cdot}75'' \times 24}{3{\cdot}1416} = 5{\cdot}72''$ or $5\frac{3}{4}''$ nearly; the diameter of B $= \frac{{\cdot}75'' \times 18}{3{\cdot}1416} = 4{\cdot}29''$ or $4\frac{5}{16}''$ nearly. From A set off AC $= \frac{1}{2}$ of $5\frac{3}{4}''$, and from C mark off CB $= \frac{1}{2}$ of $4\frac{5}{16}''$ (these dimensions being taken according to the scale of the drawing). From A and B as centres with radius AC, BC, respectively, describe the pitch circles PC. From C mark off along AB the top and bottom of the teeth of each wheel, making the top $\frac{5}{15}$, and the bottom $\frac{6}{15}$, of the pitch; through these points describe the circles *t*, *b*, for each wheel. The remaining dimensions for A are as follow:—Thickness of rim *d* $\frac{3}{8}''$; diameter of boss $2\frac{1}{4}''$, diameter of hole in boss for shaft $1\frac{1}{4}''$; key for shaft $\frac{5}{16}''$ square, fixed half in wheel and half in shaft; width of teeth *f* $1\frac{3}{4}''$; width through boss $2''$; and thickness of plate $\frac{3}{8}''$. These dimensions are usually given in terms of the pitch, to which we shall refer later on. In fig. 102 half of each wheel is in section.

50. We will give a formula which connects the three varying quantities (the pitch, number of teeth, and diameter) of the pitch circle. Let P stand for the pitch, D the diameter of the pitch circle, and N the number of teeth, P and D being given in inches and parts of an inch;—

$$\text{then } P \times N = D \times \pi \quad - \quad - \quad - \quad - \quad (1);$$

which may be put in the forms—

$$N = \frac{D\pi}{P} \quad - \quad (2), \qquad D = \frac{PN}{\pi} \quad - \quad - \quad (3).$$

If we know any two of the quantities N, P, or D, the third may be found. Equation (3) is the form most required.

51. Bevel Wheels.—If the shafts to be connected are not parallel, but lie in the same plane, bevel wheels are used to connect them. They consist of frusta of cones provided with teeth on the conical surface. We shall consider them in the first instance as toothless. First, when the shafts are at right angles:—Let it be required to connect the axes A and B (*aa*, *a'a'*, *bb*, *b'b'*), figs. 103, 104, by means of bevel wheels, so that A shall make two revolutions to one of B. Upon *b'b'*, fig. 103, set off from D along D*b'* any convenient length D1 as a unit of length; and upon *a'a'*, from D, a distance D2 equal to two of the same units of length. Upon D1, D2, describe the rectangle D1C2, and draw the diagonal DC. Let *e'f'* be the greatest radius of the driving wheel, draw *f'f* parallel to *a'a'*, meeting DC in *f*. Through *f* draw lines parallel to *a'a'* and *b'b'*; make *eg* = *ef*, and *hk* = *hf*; then *gf* and *kf* will be the required greatest diameters of the wheels. Join D*k*, D*g* (*k*D*g* will be a straight line), then D*fg*, D*fk* are two cones having a common vertex D, which, being centred upon the axes A and B (*a'a'*, *b'b'*), will revolve in contact so that the axis A shall make two revolutions while the axis B makes one. The line D*f* is the line of contact. Frusta of cones are used for the wheels, as shown in the figures. Fig. 103 is an elevation, fig. 104 a plan, of the wheels.

52. We will now extend the case to include bevel wheels whose axes are not at right angles; but, as in the former case, lying in the same plane. In figs. 105, 106, the axes A and B (*aa*, *bb*, *a'a'*, *b'b'*) are inclined at an angle of 60°, and when produced meet in a point D. It is required to connect the axes so that B shall make three revolutions to two of A, the greatest diameter of the wheel on A to be equal to *e'f'*, fig. 106. Draw the axes *b*D, D*a* to contain an angle of 60°. Upon D*a* set off D2 = 2 units of length (D1), and upon D*b* set off D3 = 3 of the same units. Upon D3, D2, describe a parallelogram D2C3, draw the diagonal DC, which is the line of contact. Draw *f'f* parallel to *aa*, meeting DC in *f*, through *f* draw *feg*, *fhk* perpendicular to *aa*, *bb*, respectively, making *eg* = *ef* and *hk* = *hf*. Join D*k*, D*g*, then D*fk*, D*fg* are two cones which are the pitch surfaces of the required wheels, as in the last example; frusta of these cones are used for the wheels. The same equations are applicable for bevel wheels as those given for spur wheels in Art. 50, page 43.

53. We will now take an example, applying it to the case of equal bevel wheels with axes at right angles, called *mitre wheels*. Fig. 107 is an elevation, and fig. 108 a plan, of a pair of mitre wheels in *gear* of 24 teeth, 1″ pitch; the diameter of pitch circle = 7·636″, or $7\frac{5}{8}''$ nearly. Draw the centre lines *aa*, *a'a'*, *bb*, *b'b'*, and the pitch circles *kf*, *fg* in plan and elevation, join C*f*, C*g*, C*k*, fig. 108. From *f* draw *fe* perpendicular to C*f*, meeting the axis *aa* in *e*, and join *eg*; then *eg* is perpendicular to C*g*. Draw similar lines from *f* and *k* for the other wheel. From *f* along *lfe* set off the top *t* and bottom *p* of the teeth of each wheel, as shown at *g*; from each of these points draw lines to C. Upon *f*C or *g*C mark off *gm*, equal the width of the teeth, making both wheels similar. The construction lines show how to complete the drawing. The teeth of bevel wheels are made of the same size as the teeth of spur wheels of the same pitch at *tgp*, fig. 108, but as they radiate to a common centre C, they decrease in size the nearer they are to that centre. The following dimensions may be added:—Diameter of hole for shaft $1\frac{1}{2}''$; diameter of boss $2\frac{3}{4}''$; width through boss $2\frac{3}{8}''$; width of teeth *gm* $2\frac{1}{2}''$; and the key $\frac{5}{16}''$ square. The other proportions of the teeth are to be taken from one of the sets of dimensions given in Art. 66, page 51.

We shall treat bevel wheels more fully in the Advanced Work of this Series.

54. THE connection between shafts by means of pulleys and bands for the transmission of motion.

Pulleys and Bands.—In the previous chapter we considered how motion could be transmitted by wheelwork; we shall now refer to a more simple and less expensive means of obtaining similar results; it is one, however, which can only be applied in cases where the motion transmitted may vary in extreme cases without causing serious inconvenience, as for instance, when one shaft turns round a fraction of a revolution more or less than the other. However, there are cases where pulleys and bands cannot be used for other reasons, but these refer more especially to the transmission of motion between shafts in machines where the space is limited, and where a direct connection is essential. The arrangement under consideration has many advantages over that of connection by wheelwork; one is that the shafts may be any distance apart within reasonable limits; and a second is, that any variation of the distance between the shafts does not alter their rate of motion, as the band can be adjusted to suit the change, and the pulleys can be made of a relative diameter, so as to produce the required number of revolutions per minute of each shaft independent of