8. In a given triangle to inscribe a circle.

If the points of contact be joined show that the triangle thus formed can be equiangular to the original triangle only in the case in which both are equilateral.

9. Show, after the manner of Euclid, that triangles are to one another in the ratio compounded of the ratios of their bases and altitudes; and prove that this is algebraically equivalent to product of ratios.

10. Similar triangles are to one another in the duplicate ratio of their homologous sides.

Two circles touch, and through the point of contact lines are drawn cutting the circles, and the ends of these lines are joined. Prove that the triangles so formed are as the squares of the diameters of the circles.

HINTS.

1. I. 32.

Sum of int. $\angle$'s. of the n Δ's $=2n$ rt. $\angle$'s.

$=$ all the vertical $\angle$'s. $+$ twice all exterior $\angle$'s. of fig.

$=$ " " " $+8$ rt. angles.

$\therefore$ Vertical angles alone$=(2n-8)$ right angles.

3. II. 11. There are five such lines.

4. Let x - part between obtuse angle and perp.

then $25^2+44^2+50x=4000$.

$\therefore$ $x=27$.

The perp. may now be found by I. 47 and thence the area.

5. Let A be the given point in the circumference. Describe another circle equal to the given circle and touching it externally at A. Produce the chords of the first circle through A to meet the circumference of the second circle, then the enclosed arcs are respectively equal. The diameters are also equal, whence it may be shown that all the other chords are respectively equal.

6. III. 21. Let CA and CB be the two fixed lines. From A draw AD perp. to AC cutting BD, the perp. from B on BC, in D. From A draw also AE perp. to BC, and from BF perp. to AC, and let these intersect at G. A circle will go round $ADBC$, III. 22. The middle point H, of CD is the centre of this circle. In the triangle AHB, the angle at $H=$half the angle C which is constant. Also the angles HAB and HBA are equal and constant. Hence when AB is constant the triangle HAB is constant, and CD or $2AH$ is constant, i.e. the point D is always in the circumference of the circle $ABCD$. Similarly G is always in the circumference of $CEFG$.

7. Draw CD perp. to AB, and it makes the angle $DCB=$angle at A, and angle $ACD=$angle at B. Through C draw CK perp. to CD. Bisect BC at right angles by QK meeting CK in K, then $CK=KB$, and a circle described through C with centre K passes through B. The angle in the segment outside the triangle$=$angle at A. Similarly the circle touching CD at C and passing through A has an external segment whose angle$=$angle at B. It is manifest that the circles touch one another at C.

8. IV. 6. Join EG, GF, BE. The angles at D are the supplements of the angles at A, B, and C; also, the angles at D are double the angles E, F, G of the triangle EFG; i.e., the angles of EFG are half the supplements of the angles of ABC. Hence unless the angles A, B, and C are equal, the angles E, F, G are not equal. And when A, B, C are equal angles, the triangles ABC and EFG must be equilateral.

10. Draw the common tangent. It is easily shown, III. 31, that the triangles are similar, and their bases parallel. Draw the diameters parallel to these bases and complete the triangles by joining the extremities with the point of contact. These triangles are similar to the first triangles. Their ares are as the squares of their bases, i.e. as the squares of the diameters. Hence the first triangles are as the squares of the diameters.

CORRESPONDENCE.

MR. J. H. THOMPSON, Moncton. Ont., asks for solutions to the following:—

1. If a shot weighing 52 lbs. be fired from a gun weighing 2 tons, with a velocity of 1,120 feet per second, and if the friction between the gun and the ground be equal to a force of 1 ton, how far will the gun recoil?

2. A body weighing 12 lbs. slides with uniform velocity down a plane that rises 5 in 13. How much energy would have to be expended in order to drag the body 13 feet up the plane by means of a string stretched parallel to the plane?

3. A ball is fired from the ground at an angle of 45°, so as just to pass over a wall 10 feet high at a distance of 200 feet. How far will it strike the ground from the wall? $g=32$.

MR A. T. DELURY, Manilla, Ont., sends us the following solution of number 6, page 5, in January number. We return cordial thanks.

Let OAB be $\triangle$ given by question. Produce OA and OB to C and D making $OC=m.OA$, and $OD=n.OB$. Then OC and OD represent the forces. Complete $\square$ $ODFC$. Then OF represents resultant. Draw CEK parallel to AB cutting OF in E and OD in K. Then OAG and OCE are similar $\triangle$'s.

$$\therefore \text{(Euc. VI. 4.)}\ \frac{OC}{OA}=\frac{OE}{OG}.$$

But $OC=m.OA$. $\therefore$ $OE=m.OG$.

Again $\angle EFC$ and GOB are equal (Euc. I. 29), and $\angle OGB=\angle AGE=\angle CEF$. $\therefore$ $\triangle CEF$ and GOB are similar.

$$\therefore \frac{CF}{OB}=\frac{FE}{OG}.\ \text{But}\ CF=OD=n.OB.\quad \therefore FE=n.OG.$$

$\therefore$ The whole $OF=(m+n)OG$.

$\therefore$ $(m+n)OG$ represents resultant.

Again. $\triangle CEF$ and OEK are similar.

$$\therefore \frac{OE}{EF}=\frac{KE}{EC}.\ \text{But}\ \frac{OE}{EF}=\frac{m.OG}{n.OG}=\frac{m}{n}.\quad \therefore \frac{KE}{EC}=\frac{m}{n}$$

And since CK is parallel to AB it may easily be proved that

$$\frac{KE}{EC}=\frac{BG}{GA}.\quad \therefore \frac{BG}{GA}=\frac{m}{n}.\quad \therefore m.GA=n.GB.$$

which shows position of OG. Hence the proposition.

MR. A. D. FRASER, P. E. I., presents a neat solution of problem 5, page 252 of December issue.

$DG^2+AD^2=AE^2+EG^2$. (I. 47 & Ax. 1.)

i.e. $DG^2+AB^2+BD^2+2AB{\cdot}BD=AE^2+EB^2+BG^2+2EB.BG$ (II. 4.)

but $DG^2+BD^2=BG^2$ and $AB^2=AE^2+EB^2$. (I. 47.)

$\therefore$ $2AB.BD=2EB.BG$.

i.e. $AB.BD=EB.BG$.

Mr. T. F. SPAFFORD, Demorestville, sends the following for solution:—

4. A sphere, diameter 4 feet, is submerged until its centre is 5 feet beneath the surface of a pond; divide the sphere into two equal parts by a horizontal plane so that the surfaces shall be equally pressed.

5. If $x=\left(\frac{a+b}{a-b}\right)^{\frac{2mn}{n-m}}$, then $\frac{1}{2}\cdot\frac{a^2-b^2}{a^2+b^2}\left(\sqrt[m]{x}+\sqrt[n]{x}\right)$

$$=\left(\frac{a+b}{a-b}\right)^{\frac{m+n}{n-m}}$$

6. Solve,—

$$\frac{xy}{4y-3x}=20;\quad \frac{xz}{2x-3z}=15;\quad \frac{yz}{4y-5z}=12.$$

C. FESSENDEN, B.A., Napanee, kindly points out that the words suggested in emendation of question 6, p. 251, of last vol., would not be an improvement. We regret that we have not more space to give the acute observations made on the problem. We hope all our correspondents will continue to let us know of their wants.

THE BOAT THE GNATS BUILD.—Did you ever hear about the wonderful boats the gnats build? They lay their eggs in the water, and the eggs float until it is time for them to hatch. You can see these little egg rafts on any pool in Summer. The eggs are so heavy that one alone would sink. The cunning mother fastens them all together until they form a hollow boat. It will not upset, even if it is filled with water. The upper end of these eggs is pointed, and looks very much like a pocket flask. One egg is glued to another, pointed end up, until the boat is finished. And how many eggs do you think it takes? From two hundred and fifty to three hundred. When the young are hatched they always come from the under side, leaving the empty boat afloat. These eggs are very, very small. First they are white, then green, then a dark gray. They swim just like little fishes, and hatch in two days. Then they change again into a kind of sheath. In another week this sheath bursts open and lets out a winged mosquito. It is all ready for work. There are so many of them born in a summer, that, were it not for the birds and larger insects, we should be "eaten up alive."—*Our Little Ones.*