principle that it must be so. It is in reality only a special case coming under a more general method; and it would therefore be more correct to work in the converse direction, and to find the depth of the spandrel and the amount of backing required to make the curve take the desired form, as nearly as the case will allow.

We may now proceed to consider the arch carrying a live load symmetrically placed. This load we may reduce in the usual way to an area representing an equivalent amount of masonry.

The load on Railway arches is now often as great or greater than the weight of the structure itself, which shows again how unsafe it is to apply methods which depend upon its amount being neglected. If the load is uniformly distributed, the centre of gravity of the whole area representing arch ring spandrel and load between the joint of rupture and the key, will almost always be further from the key than the centre of gravity of the arch ring itself; and this being the case, the reasoning given before will be entirely applicable. The joint of rupture will still have the same position as in the arch ring, and the general method as given for the full arch and the segmental arch will be the same.

The gravity line with a distributed load will be nearer the key than in the unloaded arch, and the point J will also be nearer the intrados. With an excessive distributed load, or with a load increasing towards the key (while still symmetrical as regards the two sides of the arch), the point J may reach the intrados. This is an unfavorable position, but it does not necessarily compromise the stability of the arch. It is desirable if possible to prevent its occurrence, either by thickening the arch ring, modifying the form of the arch, or re-arranging the weight in the spandrels in order to keep the gravity line further from the key.

If for any positions of the line load which the arch has to carry the gravity line is brought still nearer the key, the line $L\ J$ will continue tangent to the intrados and the joint of rupture will rise. In such extreme cases it is possible that the point K may also rise at the key, even with unyielding abutments. It appears both from experiments with models, and by comparison with the form of the funicular polygon, that the curve of pressure will rise immediately under any highly concentrated load; and with a moving load such concentration is necessarily greatest at the crown. It would therefore seem probable that the thickness of the arch ring at the crown depends more directly upon the heaviest concentrated load passing over the arch than on any other consideration. Vibration on the contrary has more effect towards the haunches.

We are still dependent, however, upon empyrical formulae for the thickness at the crown. Early authors suggested an equation of a linear form; but this has now been replaced by the forms

$$t = C\sqrt{s} \qquad \text{or} \qquad t = C\sqrt{r}$$

t being the thickness required, s the span of the arch, and r the radius at the crown, all in feet; and C a constant. (17) The values most generally adopted for C are those given by Rankine and Dupuit, which are as follows:—

Rankine. $t = \sqrt{0{\cdot}12\ r}$ for a single arch.

$t = \sqrt{0{\cdot}17\ r}$ for an arch of a series.

Dupuits. (Coefficients reduced for feet)

$t = 0.36\ \sqrt{s}$ for a full arch.

$t = 0\ 27\ \sqrt{s}$ for a segmental arch.

Rankine's introduction of the radius of curvature at the crown instead of the span, is based upon a comparison between the arch and an elastic rib; but he deduces the co-efficients from actual examples. With regard to the greater value in the case of arches in series, he considers that yielding is more likely to take place when a loaded arch stands between two unloaded ones, than if it stood between abutments. Dupuit's formulae are based directly on very numerous examples, a large proportion being Railway bridges. He refers only to actual construction as justifying the lower co-efficient he gives for segmental arches (18)

The starting point for all such formulae is the semi circular arch of diameter d; and for it a comparison between the formulae can readily be made. We have for a single arch:—

Rankine. $t = 0\ 24\ \sqrt{d}$ Dupuit. $t = 0{\cdot}36\ \sqrt{d}$

The greater value given by Dupuit's formula is probably due to the greater proportion of Railway arches on which it is based, and perhaps also to the lower average strength of the stone used in France. If we consider the span to remain the same, and the arch to change from a semi-circle to a straight arch by passing through the intermediate segmental forms, we find that Dupuit would give the arch ring a diminished thickness. Rankine's formula, on the contrary, would make it continually increase, and on arriving at the straight arch the thickness would be infinite, or in other words a straight arch would be theoretically impossible. This may result from the supposition that the joint lines are always radial; and his formula might still be applied to the straight arch by taking the point from which the joints radiate as the supposed centre. An intermediate formula has been proposed by Trautwine in the last edition of his Pocket-book:—

$$t = 0.25\sqrt{r + \tfrac{1}{2}s} + 0.20 \text{ (for feet)}$$