Mathematical Department.

TORONTO UNIVERSITY EXAMINATIONS, 1883.

FIRST EXAMINATION.

EUCLID AND TRIGONOMETRY.

Examiner—EDGAR FRISBY, M.A.

1. If a straight line be bisected, and produced to any point, the square on the whole line thus produced, and the square on the part of it produced, are together double of the square on half the line bisected, and of the square on the line made up of the half and the part produced.

2. In every triangle, the square on the side subtending an acute angle is less than the squares on the sides containing that angle by twice the rectangle contained by either of these sides, and the straight line intercepted between the perpendicular let fall on it from the opposite angle, and the acute angle.

In any quadrilateral, the squares on the diagonals are together equal to twice the sum of the squares on the straight lines joining the middle points of opposite sides.

3. The angles in the same segment of a circle are equal to one another.

Given the base AB, and vertical angle C, any line drawn dividing this angle in a given ratio will pass through a fixed point.

4. Describe an isosceles triangle having each of the angles at the base double of the third angle.

Are either of the two circles essential to this proposition?

5. If a straight line be drawn parallel to one of the sides of a triangle, it shall cut the other sides or those sides produced proportionately.

If a straight line be drawn parallel to the base of a triangle cutting off the n^{th} part of the sides; and diagonals of the remaining rhomboid are drawn, they will mutually cut off the $(n+1)^{th}$ part of these diagonals.

6. Similar triangles are to each other in the duplicate ratio of their homologous sides.

Divide a triangle into two equal parts by a straight line perpendicular to the base.

7. If from the vertical angle of a triangle a straight line be drawn perpendicular to the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained by the perpendicular and the diameter of the circle described about the triangle.

8. Prove $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80}=\log 2$.

9. The angles of a triangle are in the ratio 1:2:3 and the sum of the sides is 400 feet; find the sides.

10. The vertical angle of an isosceles triangle is 30°, and one of its equal sides is $20\sqrt{3}$ feet, find the area and the length of the perpendicular on the base.

11. The three sides of a triangle are 20, 21, and 29 feet, find the area, and the perpendicular from the opposite angle upon the longest side.

12. ABC is a triangle, and CD is drawn perpendicular to the base, show that the segments of the base are equal to

$$\frac{c^2+a^2-b^2}{2c} \text{ and } \frac{c^2-a^2+b^2}{2c}$$

SOLUTIONS.

1. Book-work. Euclid, Book II., 9.
2. (a) Book-work. Euclid, Book II., 13.

(b) Let $ABCD$ be the given trapezium. Let X be the middle pt. of AB, Y of BC, Z of CD, and W of DA. Join DB, AC, WY, XZ; also WX, YZ, WZ, XY. Then $WXYZ$ is a parallelogram. For WX, YX, &c., join the middle points of the sides of the triangles DAB, ABC, &c. Then since WX, YZ, &c., is a parallelogram, the sum of the squares on WX, XY, YZ, ZW is = the sum of the squares on WY and XZ. And doubles of these are equal. But twice the squares on WX and ZY = 4 times the square on WX = once the square on DB. Similarly for the rest.

Hence AC^2+DB^2 = twice (WY^2+XZ^2).

3. (a) Book-work. Book III., 21.

(b) Let AB be the given base and C, C_1, C_2, &c., be the given vertical angle in different positions. Now the locus of these positions is the arc of a circle which passes through A and B. Let this circle be described, and let the line CP divide the angle C in any ratio and meet the circumference again at P. Join C and P. Then the angle PC_1B = angle PCB. Hence C_1 is divided in the same ratio as C. Similarly for C_2, C_3, &c. Hence it is clear that all the lines pass through P.

4. (a). Book-work. Book IV., 10.

(b) The first and larger circle is not necessary, for we may use I. 22 to describe the triangle ABD.

5. (a) Book-work. Book VI., 2.

(b) Let ABF be the triangle. In AB let C be taken so that $AB=n.AC$. Draw CE parallel to BF, join CF and BE, cutting each other in Q. Through Q draw DQG parallel to AF. Then $BD.DA=BQ.QE$ or $FQ:QC$. It may also be shown that $AB:AC=BD:DA$. But $AB=n.AC$, $\therefore BD=n.DA$. Consequently $AB=(n+1)AD$, and therefore $BE=(n+1)QE$.

6. (a) Book-work. VI., 19.

(b) If the triangle is isosceles the perpendicular from the vertex will bisect the triangle. But if not, bisect AB in E, draw EF perp. to BC. Take BG a mean proportional between BF and BC, draw GH parallel to FE, then GH bisects the triangle.

For $\triangle BEF:BGH=BF:BC$. (Similar triangles & VI. 19.)

$$=\triangle BEF:BEC.$$
$$\therefore \triangle BGH=\triangle BEC.$$
$$=\tfrac{1}{2}\triangle BAC. \quad =HACG.$$

7. Book-work. See VI. C.

8. $7\log\frac{2^5}{3\times10}+5\log\frac{100}{2^3\times3}+3\log\frac{3^4}{2^3\times10}$

$=7(5\log2-\log3-1)+5(2-5\log2-\log3)+3(4\log3-3\log2-1)$
$=\log2.$

9. $A:B:C=1:2:3$; but $A+B+C=180°$.

$\therefore A=30°,\ B=60°,\ C=90°$;

$\therefore \sin A=\frac{1}{2},\ \sin B=\frac{1}{2}\sqrt{3},\ \sin C=1.$

Also $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$.

$\therefore \frac{1}{2a}=\frac{\sqrt{3}}{2b}=\frac{1}{c}$

and $a+b+c=400$, three equations which give a, b, and c.

10. Base angles must $=60°$. Let p = perp.
Then $\sin 60°=p\div20\sqrt{3}=\frac{1}{2}\sqrt{3}$, $\therefore p=30$.
Let b = base, then $30^2+\frac{1}{4}b^2=400\times9$, $\therefore b=60\sqrt{3}$;
and area $=\frac{1}{2}bp=900\sqrt{3}$.

11. Area $=\sqrt{(50\times49\times41)}$.
Area $=\frac{1}{2}p\times29=\sqrt{(50\times49\times41)}$, $\therefore p=$ &c.

12. Let BC be the base and p the perp. from A.
Let x = one segment, and
$\therefore a-x$ = other segment.
$\therefore c^2-x^2=b^2(a-x)^2=p^2$,
$\therefore x=$ &c.

ALGEBRA AND TRIGONOMETRY.

Examiner—W. FITZGERALD, M.A.

1. (1) Given $\left\{\begin{array}{l} x:y::a:b \\ x^2+y^2=c^2 \end{array}\right\}$ find the values of x and y.

(2) Given $\left\{\begin{array}{l} 2x+4y-3z=22 \\ 4x-2y+5z=18 \\ 6x+3y-2z=31 \end{array}\right\}$ find the values of x, y, and z.

2. Solve the following equations:

(1) $\left\{\begin{array}{r} x^2+y^2=41 \\ xy=20 \end{array}\right\}$

(2) $x^4-4x^3+6x^2-4x=15=0$

(3) $\left\{\begin{array}{l} x^2+xy+y^2=7 \\ x^4+x^2y^2+y^4=21 \end{array}\right\}$

3. Define an arithmetical and a geometrical series.

(1) Find the n^{th} term, and the sum of n terms of an arithmetical series.

(2) Insert five arithmetical means between 3 and 16.

4. In a geometrical series, if the ratio be a proper fraction, show that the sum of the series when the number of terms is increased indefinitely has a limiting value.

The limit of the sum of a geometrical series is $3\frac{1}{3}$, and the second term is $-\frac{5}{2}$; find the series.