2.5 Establish Position of Target at Time of Launch (Continued)

This may be solved using Newtonian iteration:

$$f(E_k) = E_k - e \sin E_k - M_1 = 0$$

$$f'(E_k) = 1 - e \cos E_k$$

$$E_{k+1} = E_k - \frac{f(E_k)}{f'(E_k)} = E_k - \frac{E_k - e \sin E_k - M_1}{1-e \cos E_k}$$

$$E_{k+1} = \frac{e [\sin E_k - E_k \cos E_k] + M_1}{1-e \cos E_k}$$

The first estimate of E_k may be obtained by dividing Kepler's equation into two terms.

$$M = E - e \sin E \rightarrow \sin E = \frac{1}{e} (E-M)$$

The intersection of the two lines $y = \sin E$ and $y=\frac{1}{e}(E-M)$ gives a good first estimate which results in rapid convergence. (The eccentricity of the final orbit is known.) This graphical solution is given in Figure 2-6 from the Handbook.

After convergence, this value is used in Equation 2.5-10 to give the initial position of the target satellite. The position of the target, θ_{1f}, and the corresponding time of launch measured from perigee in the final orbit, t_{1f}, are thereby established.