course to be 140 miles (L. C. M. of 28 and 35). Then, in the first case, he goes 140 miles at 8 miles an hour, 65 at 2 and 75 at 1½ m. an hour, making the whole time 100 hours. In the second case he goes 96 miles at 2 m. an hour, 44 at 1½ and 140 at 7½ m. an hour, making the whole time in this case 96 hours, or 4 hours less than before. But the difference in time is only one hour, so that we have supposed the course four times too long, consequently the length of the course is 35 miles and the shortest time 24 hours.

37. If $f(x)$ be divided by $(x-a)(x-b)$ there will be a quotient ($Q$ suppose) and a remainder of the form $cx+d$

$$\therefore \frac{f(x)}{(x-a)(x-b)} = Q + \frac{cx+d}{(x-a)(x-b)}$$

$$\therefore \frac{f(x)}{x-a} = Q(x-b) + \frac{cx+d}{x-a}$$

$$= Q(x-b) + c + \frac{ca+d}{x-a}$$

that is, the remainder, after dividing $f(x)$ by $x-a$ is $ca+d$,

$$\therefore ca+d = R$$

Similarly $cb+d = S$

$$\text{whence } c = \frac{R-S}{a-b},\quad d = \frac{Sa-Rb}{a-b}$$

$$\therefore cx+d = \frac{R-S}{a-b}x + \frac{Sa-Rb}{a-b}$$

38. Suppose the green is worth 99c. and the black 77c. per lb. Then, if they are mixed in the ratio of 5 of green to 2 of black, one pound of the mixture is worth

$$\tfrac{5}{7} \text{ of } 99 + \tfrac{2}{7} \text{ of } 77 = 92\tfrac{5}{7}.$$

Now, since he intends to increase his selling price from 110 to 121, that is from 10 to 11, without changing the selling price per lb., he must reduce the value of each lb. sold from 11 to 10, that is to $\frac{10}{11}$ of $92\frac{5}{7}$c., which $= 84\frac{2}{7}$c. He has therefore to mix tea worth 99c. with tea worth 77c., so as to form a mixture worth $84\frac{2}{7}$c. per lb. Hence he must take 51 lbs. of green tea and 103 of black.

Or thus:

Take 770 lbs. of green and 308 of black. This is worth 1298 lbs. of black, and we require a mixture containing the same number of lbs. and worth 1180 of black. If 7 lbs. of black be substituted for 7 of green, the value of the mixture will be reduced by 2 of black, hence to reduce the value by 118 we must take out $59 \times 7$ of green and replace by $59 \times 7$ of black; we shall then have 357 of green and 721 of black: these are in the ratio 51 : 103. It will be noticed that 770 and 308 are as 5 : 2 and are multiples of 2, 7 and 11.

39. $$2x\sqrt{1-x^4} = a(1+x^4)$$

Square and divide by $a^2x^4$; then

$$\frac{4}{a^2x^2} - \frac{4x^2}{a^2} = \frac{1}{x^4} + 2 + x^4$$

$$\therefore \left(x^2 - \frac{1}{x^2}\right)^2 + \frac{4}{a^2}\left(x^2 - \frac{1}{x^2}\right) + 4 = 0$$

$$\therefore x^2 - \frac{1}{x^2} = -\frac{2}{a^2}\left(1 \mp \sqrt{1-a^4}\right)$$

$$= 2k^2 \text{ suppose}$$

$$\therefore x = \pm\sqrt{\left(k^2 \pm \sqrt{1+k^4}\right)}$$

$$= \pm\frac{1}{a}\sqrt{\left\{(\sqrt{1+a^2}-1)(\sqrt{1-a^2}+1)\right\}}$$

40. The one reaches its destination in 25 minutes and the other in 100 minutes after they meet, and since these numbers are in the ratio of 1 : 4, therefore the rates of the trains are as 1 : 2; and the distances they have gone when they meet must be in the same ratio, therefore the faster train runs the remaining one-third of the distance in 25 minutes, and consequently the whole distance in 75 minutes; therefore the trains started at 11hr. 35′.

41. It will be seen that B walks the same distance each day, namely: $\frac{1}{n+1}$ of the whole distance, and therefore in $n$ days he will have gone $\frac{n}{n+1}$ of the whole distance. He could complete the journey in one day more.

A walks one-half the distance the first day, one-sixth the distance the second day, one-twelfth the third day, one-twentieth, one-thirtieth, &c., the fourth, fifth, &c., days We have therefore to sum the series.