The initial axial compression holds in equilibrium the stresses due to 42·5 per cent. of the total head at the bottom, the remaining 57·5 per cent. of the load will divide between cantilever arch and curved beam action in proportion to their relative carrying capacity.

By analyzing (6) it is seen that by simply varying t or R_u, or both, the designer can utilize more or less of the initial stress to carry the load. If the base thickness in Fig. 3 is increased from 70 feet to 110 feet and the thickness increased correspondingly at higher elevations, the initial stresses will be able to support at the foundation $0{\cdot}4 \times H \times \frac{110}{75} = 0{\cdot}585 \times H$, or 58·5 per cent. of the total water pressure before any shortening in the length of the arch occurs and before additional axial compression is introduced.

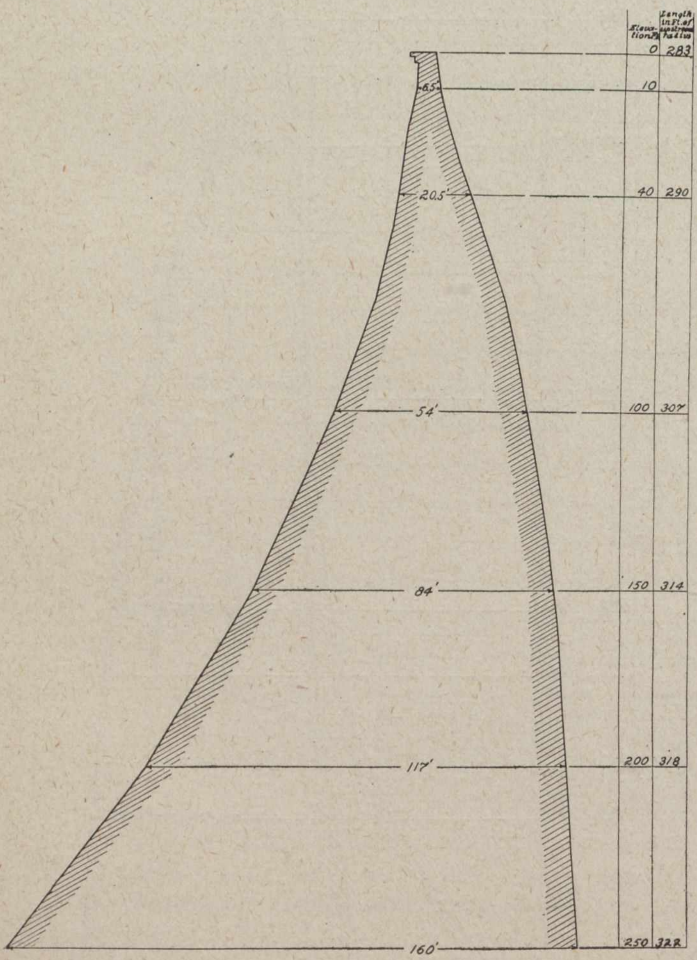

Fig. 4.

When the arch, however, becomes very thick in comparison with its length, the load is carried more by curved beam action than by ordinary arch action.

The dam shown in Fig. 3 was designed with varying radii to keep the central angle of the arch as nearly constant as possible at all elevations. For comparison, a section is shown in Fig. 4, using the same unit compression except where the section is wider than a gravity section near the foundation and the same upstream face batter, but a single common centre as ordinarily used for both upstream and downstream faces. For this section the length of the upstream radius is also variable, but it increases towards the bottom and reaches here a value of 322 feet. (See tables of lengths, Figs. 3 and 4.) The initial stresses in this dam will resist 20% of the head of the water at the bottom. It is, therefore, easily seen that the constant angle arch is much more effective in utilizing the initial stresses to carry the load than is the ordinary arch dam struck from a single centre. If a gravity section is insisted upon for the arch, but the central angle kept as near constant as practicable it will be possible for the gravity section to take up the greater part of the load acting as an arch and curved beam. The factor of safety has thereby been largely increased.

For finding the arch deflection the following formula has been used:

$$D_o = \frac{C\,C_o \times P_1\,(\text{upstream radius})^2}{E\,t} \qquad (7)$$

where $P_1 = p\,\frac{R_u}{R_m}$, and $C\,C_o$ is a factor which takes the curved beam action into consideration and can be directly found from Fig. 5.*

Formula (7) has been used for finding the deflection curves A and B (Fig. 6) of the two sections, Fig. 3, (base 110 feet), and Fig. 4. Formula (6) has been used for correcting these curves A and B to take the effect of lateral strain into consideration. These curves represent the deflection of the two arches assuming they are free to move at the foundation. They are plotted to show how evenly the deflection, curve A, slants from a maximum near the top to nearly nothing at the bottom in the constant angle arch type, Fig. 3, and how little the slant, curve B, amounts to in the ordinary type of arch dam, Fig. 4. These curves also show very plainly that from the common arch type much arch action towards the bottom cannot be expected; cantilever and beam action must take the load since no such deflection as 0·2624 in. could be possible at the point where the arch is fastened to the rock foundation. The constant angle arch type for this particular site requiring only 0·0083 in. deflection, 31·5 times less to support the same load will take most of the load upon itself acting as an arch.

For dam sites where the abutments are close together towards the foundation and where t is large compared with R_u, (7), gives the values for the crown deflection which are too large, even assuming that the dam is entirely free to move at the bottom. While this formula considers the curved beam action, it is at the same time understood that arch action is complete. However, where the arch is thick and the distance between the abutments short, the arch becomes a wedge and the horizontal curved beam takes the greater proportion of the load, as acting in this manner the support of the same load will require a smaller deflection. The deflection in the middle of a beam 1 foot wide held at both ends and uniformly loaded is:

$$D_b = \frac{P l^4}{384\,E\,J} \qquad (7\text{A})$$

The notations are the same as before, P being the unit water pressure, l the length of the beam, E the modulus of elasticity of concrete and J the moment of inertia using like units.

Whenever (7A) gives smaller values than (7) it is indicated that arch action is incomplete. The curved beam action tends to introduce axial tension along the downstream face in the middle and along the upstream face near the abutments, but the axial compression due to partial arch action and lateral expansion (Poisson's ratio) will or should much more than compensate for this tendency. If it does not, the design should be changed.

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*This Fig. and Formula (7) are reproduced from a discussion by Mr. Shirreff of a paper entitled, "Lake Cheesman Dam and Reservoir," Transaction American Society Civil Engineers, 1904, page 89. E is the modulus of elasticity and t is the thickness.