multiplicand what you would have to do with unity in order to get the multiplier.

A much less easy case occurs when we consider the solution of quadratic equations, as imaginary quantities come on the scene asking to be recognized. The recognition was conceded, but these imaginary quantities remained for many years perfectly without meaning, till, as Dean Peacock tells us in the Preface to his "Algebra," in its first form, 1830, the first attempt he could find of the interpretation of $\sqrt{-1}$ was given by M. Buée in the Philosophical Transactions for 1806. It was very imperfect, giving, however, the idea of perpendicularity. A much more thorough interpretation was given in Warren's "Treatise on the Geometrical Representation of the Square Roots of Negative Quantities," in 1828.

To recapitulate, then, the view generally accepted is that certain laws are suggested by our experience of numbers, which are applicable, with limitations, both to arithmetic and geometry, and that these laws, when made to apply to symbols without limitation, lead to results which we may not be able to interpret, but which, when capable of interpretation, are productive of new mathematical truths of the greatest importance. As Mr. George B. Halstead says, in an essay on "Algebras, Spaces and Logics," in the seventeenth volume of the *Popular Science Monthly*, New York, 1890: "An algebra is an abstract science or calculus of symbols combined according to defined laws." Here the indefinite article, *an* algebra, suggests that there may be other sets of defined laws than those derived from number; and, in fact, in the algebra of quaternions, Sir William Hamilton threw overboard the commutative law, so far as certain symbols denoting vectors were concerned, and *uv* was no longer equal to *vu*, but was equal to minus *vu*. Merely noticing, by the way, this other algebra, and for the future using the word *algebra* to mean exclusively the one the laws of which are derived from number, we may say that arithmetic suggests, algebra combines, and finally arithmetic and geometry interpret, so far as their limited power extends, and the limited state of human reason and experience allows. Professor Sylvester, in his address to the British Association in 1869, after pleading for the use of the term *mathematic* instead of *mathematics*, just as we say *arithmetic*, and not *arithmetics*, says:—

"Time was when all the parts of mathematic were dissevered, when algebra, geometry and arithmetic either lived apart or kept up cold relations of acquaintance, confined to occasional calls upon one another; but that is now at an end; they are drawn together, and are constantly becoming more and more intimately related and connected by a thousand fresh ties, and we may confidently look forward to a time when they shall form but one body and one soul."

Before concluding this description of the road by which we have arrived at the modern idea of algebra, there is one point still unsettled, which it is necessary to notice. The symbal $\sqrt{a}$ presented no difficulty to the arithmetical algebraists, for, having been defined as representing the quantity which, when multiplied by itself, gave *a*, it could, with their views, have only one value. They would say that there was but one square root of 9, and that was 3. Those on the other hand who recognized negative quantities, and the laws according to which they are combined, had to admit that 9 had two square roots, plus 3 and minus 3. The question, therefore, arose whether $\sqrt{9}$ was to represent two things or one thing. The general succession of leading text-