

Then, if R_1 be the fundamental element of the root of a pure uni-serial Abelian equation of the n^{th} degree, it will be found that

$$R_1 = A_1^n (P_m^n \phi_\sigma^n \psi_\tau \dots X_\beta^n F_\beta^n), \quad (104)$$

A_1 being a rational function of w .

The Root Constructed from its Fundamental Element.

§ 48. From R_1 , as expressed in (104), derive R_0, R_2 , etc., by changing w into w^0, w^2 , etc. Then, assuming that the root of the pure uni-serial Abelian equation $f(x) = 0$ of the n^{th} degree is

$$R_0^{\frac{1}{n}} + R_1^{\frac{1}{n}} + R_2^{\frac{1}{n}} + \dots + R_{n-1}^{\frac{1}{n}}, \quad (105)$$

what we have to do in order to construct the root is to determine what values of $R_0^{\frac{1}{n}}, R_1^{\frac{1}{n}}$, etc., are to be taken together in (105).

§ 49. From (104) we have

$$R_0 = A_0^n (P_0^m \phi_\sigma^n \dots F_0^n). \quad (106)$$

By § 8, ϕ_0 is the s^{th} power of a rational quantity. Therefore, because $s\sigma = n$, ϕ_0^n is the n^{th} power of a rational quantity. In like manner each of the expressions ψ_0^n, F_0^n , etc., is the n^{th} power of a rational quantity. And, because P_m is of the same form with the fundamental element of the root of a pure uni-serial Abelian quartic, P_0 is the fourth power of a rational quantity. Therefore, since $n = 4m$, P_0^m is the n^{th} power of a rational quantity. Therefore, from (106), R_0 is the n^{th} power of a rational quantity, and $R_0^{\frac{1}{n}}$ has a rational value.

§ 50. Let the numbers not exceeding n that measure n , unity not included, be n, y , etc. (107)

For instance, if $n = 4 \times 3 \times 5 = 60$, the series (107) is

$$60, 30, 20, 15, 12, 10, 6, 5, 4, 3, 2.$$

The n^{th} roots of unity distinct from unity are made up of the primitive n^{th} roots of unity, the primitive y^{th} roots of unity, and so on. For instance, when $n = 60$, the fifty-nine n^{th} roots of unity distinct from unity are the sixteen primitive 60^{th} roots of unity, and the eight primitive 30^{th} roots of unity, and the eight primitive 20^{th} roots of unity, and the eight primitive 15^{th} roots of unity, and the four primitive 12^{th} roots of unity, and the four primitive 10^{th} roots of unity, and the two primitive 6^{th} roots of unity, and the four primitive 5^{th} roots of unity, and the two primitive 4^{th} roots of unity, and the two primitive 3^{d} roots of unity, and