SCHOOL WORK.

MATHEMATICS.

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PROBLEMS FOR JUNIOR MATRICULATION, 1887.

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1. From the obtuse angle of a triangle draw a straight line to the base, such that it may be a mean proportional between the segments of the base.

1. About ABC describe a circle; join A the obtuse angle to D the centre; on AB as diameter describe a circle, cutting BC in F; join AF and produce to G; then $GF=FA$ $\therefore BF \,.\, FC=AF^2$ (III. 35). $\because BF : FA :: FA : FC$. Q.E.D.

2. $ABCD$ is a trapezoid, AD parallel to BC, show that $AC^2 \sim BD^2 : AB \sim CD^2 = BC+AD : BC \sim AD$.

2. Draw BE, CF at right angles to AD; then $AC^2=AD^2+DC^2+2AD \,.\, DF$, and $BD^2=AB^2+AD^2+2AD \,.\, AE$.

$\therefore AC^2-BD^2=DC^2-AB^2+2AD\,(DF-AE)$, but $DC^2-AB^2=DF^2-AE^2$.

$\therefore AC^2-BD^2=(DF-AE)\,(DF+AE+2AD)=(DF-AE)(BC+AD)$.

$\therefore AC^2-BD^2 : DC^2-AB=(DF-AE)(BC+AD) : DF^2-AE^2=BC+AD : BC-AD$. Q.E.D.

3. $AB \,.\, AC$ are two tangents to a circle; PQ a chord of the circle which produced, if necessary, meets the straight line joining the middle points of AB, AC at R; show that the angles RAP, AQR are equal.

3. M and N the middle parts of AB, AC; G the centre of the circle; RL a tangent; GA cuts BC in K and MN in F, then

$$\begin{aligned} RL^2 &= RG^2-LG^2=RF^2+GF^2-LG^2 \\ &= RA^2+GF^2-FA^2-LG^2 \\ &= RA^2+(GF+FA)(GF-FA)-LG^2 \\ &= RA^2+GA \,.\, GK-LG^2=RA^2. \end{aligned}$$

But $PR \,.\, RQ=RL^2=RA^2$,

$\therefore AQR$ and APR are similar.—Q.E.D.

4. The side AB of a triangle ABC is touched by the inscribed circle at D, and by the escribed circle at E; show that the rectangle contained by the radii is equal to the rectangle $AD \,.\, DB$ and to $AE \,.\, EB$.

4. F and R centres of inscribed and escribed circles, L, K and M, N where they touch AC and CB, and AC, CB produced; then $LM=KN$. $\therefore LA+AM=KB+BN$. $\therefore AD+AE=BE+BD$. $\therefore 2AE+ED=2BD+ED$. $\therefore AE=BD$.

Again, angle FAR= right angle, $\therefore$ angle FAD = angle ERA. $\therefore FDA$ and ARE are similar. $\therefore FD \,.\, ER=AD \,.\, AE=AD \,.\, DB=AE \,.\, EB$. Q.E.D.

5. ABC is a triangle having a right angle at C; $ABDE$ is the square described on the hypotenuse; F, G, H, are the points of intersection of the diagonals of the squares on the hypotenuse and the sides; show that the angles DCE, GFH are together equal to a right angle.

5. G the centre of square $ACKL$, H of $CBMN$; join MA, LB; then MAB, HBF, CBD are similar. $\therefore$ angle $MAB=HFB$ and angle AMB = angle DCB. But $MAB+AMB=CAB$, $\therefore DCB+HFB=CAB$, also $GFA+ACE=ABC$. $\therefore DCB+HFB+GFA+ACE$ = a right angle, $\therefore ECD+GFH$ = a right angle. Q.E.D.

6. If

$$\frac{a+(a+y)x+(a+2y)x^2+\ldots\text{ad. inf.}}{a+(a-y)x+(a-2y)x^2+\ldots}=b$$

and if x receive values in H. P., show that the corresponding values of y will be in A. P.

6. The numerator $=a+ax+ax^2+\ldots+yx+2yx^2+3yx^3+\ldots=\dfrac{a}{1-x}+\dfrac{yx}{(1-x)^2}$,

the denominator $=\dfrac{a}{1-x}-\dfrac{yx}{(1-x)^2}$,

$\therefore \dfrac{a(1-x)+yx}{a(1-x)-yx}=b$. This takes the form $\dfrac{p}{x}+q=y$. If x receives values $\dfrac{1}{k}$, $\dfrac{1}{2k}\ldots$ y becomes $pk+q$, $2pk+q\ldots$