The following nomenclature will be used throughout the paper:

H = head of water above top of pipe.

r = radius of circular pipe.

M = bending moment in pipe.

M_B = bending moment in pipe at top.

M_D = bending moment in pipe at bottom.

M_{max} = bending moment in pipe at maximum point on side.

T = tension in pipe.

T_B = tension in pipe at top.

T_D = tension in pipe at bottom.

ϕ = angle to different points (expressed in radians).

ϕ_{max} = angle to maximum bending moment point on side.

ϕ_U = angle to upper node point (point of no bending moment).

ϕ_L = angle to lower node point (point of no bending moment).

γ = weight of cubic foot of water.

J = shear at any point.

J_B = shear at top.

J_D = shear at bottom.

d_p = arm of bending moment.

d_o = arm of the bending moment at top of pipe.

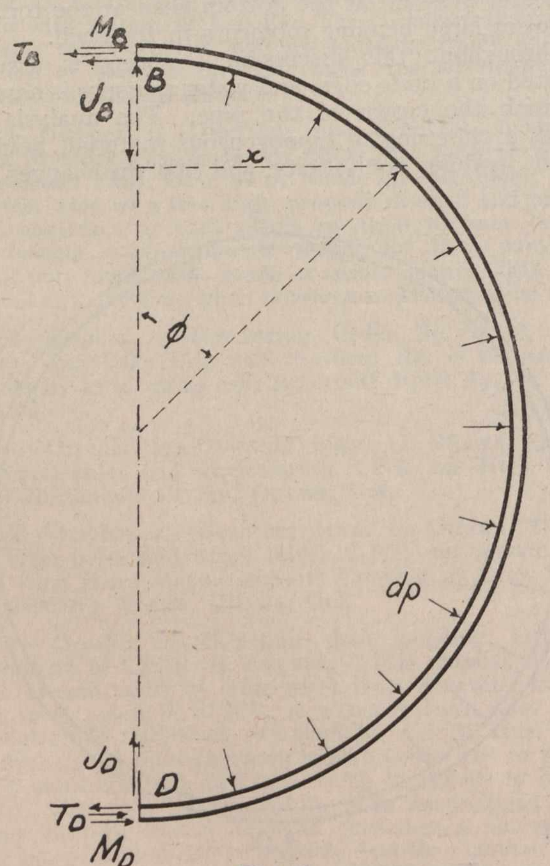

Fig. 2.

We will assume that the reader is familiar with the three general formulae for arch ribs, viz.,

$$\int_D^B \frac{Mds}{EI} = \int_D^B d\phi; \qquad \int_D^B \frac{Myds}{EI} = \int_D^B dx;$$

$$\int_D^B \frac{Mxds}{EI} = \int_D^B dy.$$

The ring is a continuous curved beam to which these situations will apply.

The forces acting upon the pipe may be appreciated by looking at Fig. 1. If we cut the pipe at B and D and consider the forces acting on the section to the right, we obtain the system of forces shown in Fig. 2.

Taking the centre of moments at the neutral axis at D, we obtain:

$$M_D = M_B - 2T_Br + \int_o^\pi r(1 - \cos\phi)\, dp \sin\phi$$

Now,

$dp = \gamma r^2 (1\text{-}\cos\phi)\, d\phi$; and,

$\sin\phi\, dp = \gamma r^2 (1\text{-}\cos\phi) \sin\phi\, d\phi$.

Therefore,

$$M_D = M_B - 2T_Br + \int_o^\pi \gamma r^3 (1\text{-}\cos\phi) \sin\phi\, d\phi.$$

Integrating, we obtain:

$$M_D = M_B - 2T_B r + 2\gamma r^3.$$

Now, consider as a free body that portion of pipe shown in Fig. 3.

$$EI\Delta y = \int Mxds;\ ds = rd\phi = rd\theta;$$

$$-M = M_B - T_By + \int_o^B pdsx;$$

$$pds = \gamma r^2 (1\text{-}\cos\theta)\, d\theta;$$

$$x = r \sin(\phi - \theta);$$

$$y = r(1\text{-}\cos\phi).$$

Therefore,

$$-M = M_B - T_Br (1\text{-}\cos\phi) + \int_o^\phi \gamma r^3 (1\text{-}\cos\theta) \sin(\phi-\theta)\, d\theta;$$

$$= M_B - T_Br + T_Br\cos\phi + \gamma r^3 \int_o^\phi (1\text{-}\cos\theta)\sin(\phi-\theta)\, d\theta,$$

$$= M_B - T_Br + T_Br\cos\phi + \gamma r^3 (1\text{-}\cos\phi - \tfrac{1}{2}\phi\sin\phi).$$

Therefore,

$$EI\Delta y = \int Mxds = \int Mr^2 \sin\phi\, d\phi,$$

$$= \int_o^\pi \Big[(M_Br^2 - T_Br^3 + \gamma r^5) \sin\phi\, d\phi + (T_Br^3 - \gamma r^5) \sin\phi\cos\phi\, d\phi - \tfrac{1}{2}\gamma r^5 \phi \sin^2\phi\, d\phi \Big]$$

Solving the above equation we obtain:

$$EI\Delta y = -2(M_Br^2 - T_Br^3 + \gamma r^5) + \frac{\gamma r^5 \pi^2}{8};$$

also, $EI\Delta x = \int_D^B M(ds)y = 0$, since B has not moved horizontally, and since its tangent is horizontal.

Now, $y = r(1\text{-}\cos\phi)$; $ds = rd\phi$.

Therefore, $EI\Delta x = r\int_o^{\pi r} Mds - r^2\int_o^\pi M\cos\phi\, d\phi$

i.e., $0 = r\int_o^{\pi r} Mds - r^2\int_o^\pi M\cos\phi\, d\phi$.

Substitute value of M found above and solve.

Therefore, $-\int_o^\pi M\cos\phi\, d\phi = \int_o^\pi \Big[(M_B - T_Br + \gamma r^3) \cos\phi\, d\phi + (T_Br - \gamma r^3)\cos^2\phi\, d\phi - \tfrac{1}{2}\gamma r^3 \phi\sin\phi\cos\phi\, d\phi \Big]$.

Therefore, $-\int_o^\pi M\cos\phi\, d\phi = \frac{\pi}{2}\left\{ T_Br - \gamma r^3 \right\} + \frac{\gamma r^3 \pi}{8}$;

or, $\int_D^B Myds = \frac{\pi r^3}{2}\left\{ T_B - \frac{3\gamma r^2}{8} \right\} = 0$;

and, therefore, $T_B = \frac{3}{4}\gamma r^2$.