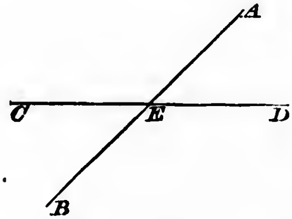
Proposition XV. Theorem.

If two straight lines cut one another, the vertically opposite angles must be equal.



Let the st. lines AB, CD cut one another in the pt. E.

Then must $\angle\ AEC = \angle\ BED$ *and* $\angle\ AED = \angle\ BEC$.

For $\because$ AE meets CD,

$\therefore$ sum of $\angle$s AEC, AED = two rt. $\angle$s. I. 13.

And $\because$ DE meets AB,

$\therefore$ sum of $\angle$s BED, AED = two rt. $\angle$s; I. 13.

$\therefore$ sum of $\angle$s AEC, AED = sum of $\angle$s BED, AED;

$\therefore$ $\angle\ AEC = \angle\ BED$. Ax. 3.

Similarly it may be shewn that $\angle\ AED = \angle\ BEC$.

Q. E. D.

Corollary I. From this it is manifest, that if two straight lines cut one another, the four angles, which they make at the point of intersection, are together equal to four right angles.

Corollary II. All the angles, made by any number of straight lines meeting in one point, are together equal to four right angles.

Ex. 1. Shew that the bisectors of AED and BEC are in the same straight line.

Ex. 2. Prove that $\angle\ AED$ is equal to the angle between two straight lines drawn at right angles from E to AE and EC, if both lie above CD.

Ex. 3. If AB, CD bisect each other in E; shew that the triangles AED, BEC are equal in all respects.