parallel to the forces through the point in the beam where the deflection is wanted. If the deflection is wanted at some place $x$ distant from the wall, then only the bending moment area between the wall and the point $x$ is used.

If the loading on the cantilever is very irregular so that the algebraic calculation of bending moment would be tedious or perhaps impossible, the graphical method may be employed. The free end of the beam may be taken at the left end, and the load diagram is broken into strips of equal width, as was done in Fig. 1. Each force is assumed to act at the middle of each strip, and a funicular and force polygon are drawn. The moment at any distance from the free end will be proportional to the intercept on a line through this point parallel to the forces, of the first and last strings for all the forces to the left of the point in question, or in other words, to the intercept of the funicular polygon. As in Fig. 1

$$M = Y_1 L H_1 F$$

Fig. 3 shows a numerical case illustrating a beam overhanging its ends. A simple loading was chosen so

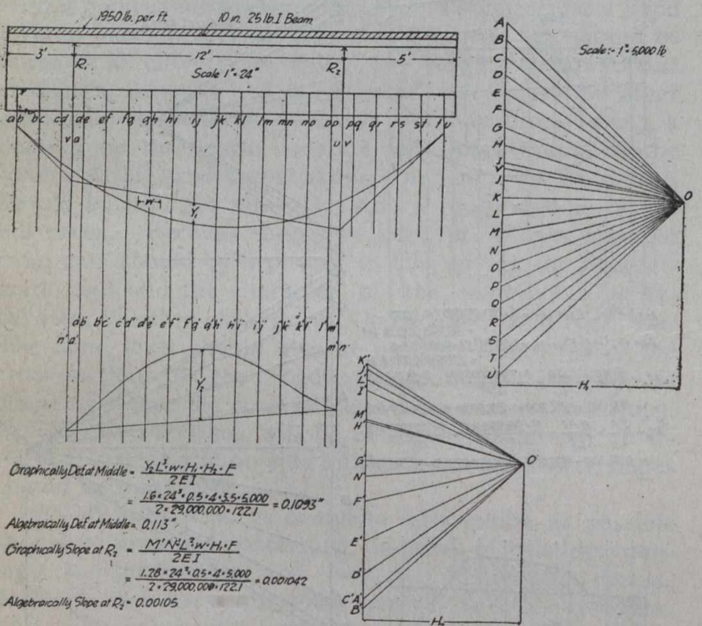


Fig. 3

that the results could be checked algebraically. If we consider only the bending moment diagram between supports and treat this as a load on a beam, a funicular polygon may be drawn which will have ordinates which are proportional to the deflections. Fig. 3 makes the method plain.

For deflections of the overhanging ends, the moment of the bending moment area will be involved as in the case of a cantilever, but here must be taken into consideration the fact that the slope at the two supports is not zero. As shown in Fig. 3, the slope at the supports is proportional to the reaction at each support as determined by the second force polygon in this figure.

$$\text{Slope at right support} = \frac{M'N'L^2 wH_1F}{EI}$$

Here the notation is the same as before and $M'N'$ is the reaction at the right support in inches, scaled from the second force polygon. In the figure the 2 appearing in the denominator of the equation is due to the fact that in drawing the second force polygon the $Y_1$ lengths in the first funicular polygon were doubled.

It can be seen that if the slope at the right support is positive the effect on the beam will be to reduce the deflection due to the overhanging end, while negative slope would increase the deflection. We know that in a simple beam treating positive bending moment area as a downward load on the beam we would get upward or positive reactions and the slope at the right support would be positive. So, if on this basis the reaction at the right support is positive, we know the slope is positive. Conversely, if positive moment is treated as an upward force, then the reaction at the right support would be negative, and then negative reaction would mean positive slope.

For the part of the beam overhanging at the left end we can see that here positive slope would increase the deflection and negative slope would decrease the deflection of the overhanging end.

It can be seen in Fig. 3 that the first funicular polygon would not give very accurate values of moment. It is better, therefore, to proceed as in the case of the cantilever beam, calculate the moments in the overhanging part algebraically, and thus determine the moment diagram. The moment of this area with respect to the free end is then found graphically, as shown in Fig. 4. This quantity

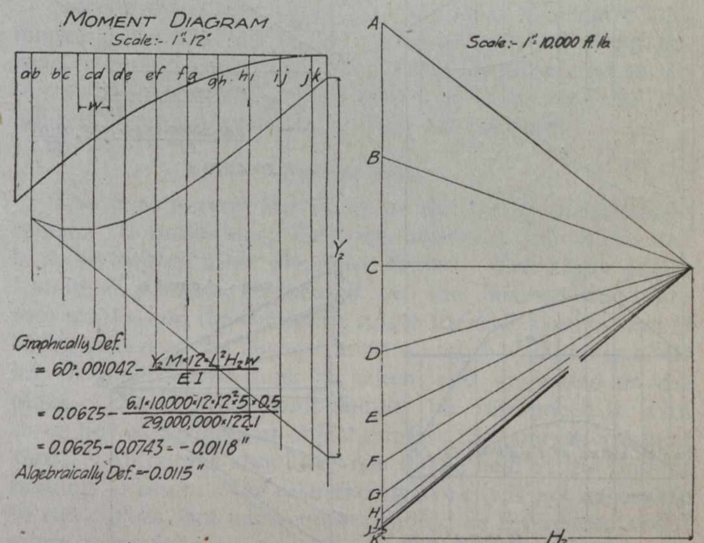


Fig. 4

must have added to it or subtracted from it the product of the slope at the right support times the distance from the right support to the free end of the beam. Since the slope at the high support is positive the effect is to decrease the deflection. Thus the product is subtracted, as shown in Fig. 4.

If the deflection is wanted at any point $x$ distant from the right support, the product of the slope times the distance $x$ is added or subtracted from the moment of the bending moment diagram from the right support to the point $x$, with respect to the section at $x$.

Fig. 5 shows the case of a beam fixed at one end and supported at the other. To determine deflections graphically probably the best way to proceed is to determine $R$, the reaction at the supported end, and then calculate algebraically the moments, say, for every foot of length. The moment diagram can then be plotted and from this a deflection curve may be obtained, as was done in the case of the overhanging beam between supports.

Since in the case before us the supported end of the beam does not deflect, the deflection caused by $R$ must be exactly equal to the deflection caused by the loads on the beam. Or, since the deflection of a simple cantilever beam at the free end is proportional to the moment of the bending moment area with respect to the free end, it follows that the moment of the bending moment area due