

11. Divide a given straight line into two parts, so that the rectangle contained by the whole and one of the parts shall be equal to the square on the other part.

12. Define the terms, *circle*, *tangent to a circle*, and *segment of a circle*.

The angles in the same segment of a circle are equal to one another.

PROBLEMS.—HONORS.

1. If a straight line terminated by the sides of a triangle be bisected, no other line terminated by the same two sides can be bisected in the same point.

2. If two equal circles be described cutting each other in A and B , and from A a chord be drawn cutting them in C and D , prove that the part CD between the circumferences will be bisected by the circle described on AB as diameter.

3. Circles are described on two of the sides of a triangle as diameters, and each meets the perpendicular from the opposite angular point on its diameter in two points; prove that these four points lie on a circle whose centre is at the intersection of the two sides.

4. Prove that

$$a^2 \left(\frac{1}{b} - \frac{1}{c} \right) + b^2 \left(\frac{1}{c} - \frac{1}{a} \right) + c^2 \left(\frac{1}{a} - \frac{1}{b} \right) \\ a \left(\frac{1}{b} - \frac{1}{c} \right) + b \left(\frac{1}{c} - \frac{1}{a} \right) + c \left(\frac{1}{a} - \frac{1}{b} \right) \\ = a + b + c.$$

5. If $x + y + z = xyz$ prove that

$$\left(\frac{x}{y} + \frac{y}{x} + \frac{y}{z} + \frac{z}{y} + \frac{z}{x} + \frac{x}{z} + 2 \right)^2 \\ = (1 + x^2)(1 + y^2)(1 + z^2).$$

6. Solve the equations

$$x + y + z = 2(a + b + c), \\ ax + by + cz = 2(ab + bc + ca) \\ (b - c)x + (c - a)y + (a - b)z = 0.$$

7. A waterman rows a given distance a and back again in b hours, and finds that he can row c miles with the stream in the same time as d miles against it. Find the time each way, and the rate of the stream.

8. ABC is an isosceles triangle, D the middle point of the base BC . If any straight line drawn through D meets one side in E

and the other produced in F , then AE, AC, AF are in harmonic progression.

9. Given

$$\tan^2 x + \sec 2x = \frac{7\sqrt{3} - 10}{\sqrt{3}} \text{ find } x.$$

10. If $A^1 B^1 C^1$ be the angles which the sides of a triangle subtend at the centre of the inscribed circle, prove

$$4 \sin A^1 \sin B^1 \sin C^1 = \sin A + \sin B + \sin C.$$

$$11. \text{ If } \cos^2 \theta = \frac{\cos a}{\cos \beta}, \quad \cos^2 \theta^1 =$$

$$= \frac{\cos a^1}{\cos \beta} \text{ and } \frac{\tan \theta}{\tan \theta^1} = \frac{\tan a}{\tan a^1},$$

$$\text{prove that } \tan \frac{\beta}{2} = \tan \frac{a}{2} \tan \frac{a^1}{2}.$$

12. If $\cos \theta = \tan \lambda \cot a$, $\cos \phi = \tan \lambda \cot \beta$, and $\sec \theta \sec \phi = \sec \lambda \tan \theta \tan \phi - \tan a \tan \beta$; shew that $\cos^2 \lambda = \cos^2 a \cos^2 \beta$.

13. Four points, moving each at a uniform speed, take 198, 495, 891, 1155 seconds respectively to describe the length of a given straight line. Supposing them to be together at any instant at the same end of the line, and to move in it from end to end continually, what interval of time will elapse before they are together at the same point again.

TRIGONOMETRY.—HONORS.

1. Define a logarithm. Of what two parts is a logarithm composed? Shew that in the common system, one of these parts may be determined by inspection.

$$\text{Prove that } \log \frac{m}{n} = \frac{m}{n} \log x.$$

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

2. Write down the characteristics of the common logarithms of 0.2, 0.00005, and 5555.5.

3. Find the logarithms of

$$\frac{\sqrt{3} \cdot \sqrt[3]{282.9}}{7 \cdot \sqrt{7.2798}}, \quad \frac{\sqrt{.003}}{2 \cdot \sqrt[3]{.05}}, \quad \sin 60^\circ, \\ \cot 45^\circ, \quad \operatorname{cosec} 30^\circ.$$

Find x from the equation $(1.08)^x = 2$.

4. Define the following trigonometrical ratios of any angle, viz., the sine, the cosine,