

$$\frac{a\sqrt{b}+b\sqrt{a}}{\sqrt{a}+\sqrt{b}} = \frac{(a\sqrt{b}+b\sqrt{a})(\sqrt{a}-\sqrt{b})}{a-b}$$

$$= \frac{\sqrt{ab}(a-b)}{a-b} = \sqrt{ab},$$

$$\{\sqrt{(4+3j)} + \sqrt{(4-3j)}\}^2 = 8 + 2\sqrt{19},$$

$$\left(\frac{-1+j\sqrt{3}}{2}\right)^2 + \frac{-1+j\sqrt{3}}{2} + 1 = 0.$$

6. Given the first term, the common difference and the number of terms of an arithmetical progression, find (1) the sum of the terms, (2) the sum of the squares of the terms.

(1) Book work.

$$(2) S = a^2 + (a+d)^2 + \dots + \{a + (n-1)d\}^2$$

$$= na^2 + d^2(1^2 + 2^2 + \dots + n^2 - 1^2)$$

$$+ 2ad(1 + 2 + \dots + n - 1)$$

$$= na^2 + \frac{n(n-1)(2n-1)}{6}d^2 + n(n-1)ad.$$

7. Solve the equations

(1) $(a-x)^2 = (x-b)^2$;

(2) $ax + by = \frac{a}{x} + \frac{b}{y} = 1$;

(3) $x(y+z^{-1}) = u$, $y(z+x^{-1}) = b$, $z(x+y^{-1}) = c$.

8. What value (other than 1) must be given to q that one of the roots of $x^2 - 2x + q = 0$ may be the square of the other.

If a, b, c are the roots of $x^3 - px^2 + qx - r$, express

$$\frac{2a^2b^2 + 2b^2c^2 + 2c^2a^2 - a^4 - b^4 - c^4}{2ab + 2bc + 2ca - a^2 - b^2 - c^2}$$

in terms of p, q and r .

If $x^2 - 2x + q = (x-a)(x-a^2)$

$$a + a^2 = 2$$

$$a^3 = q$$

$$\therefore q^{\frac{1}{3}} + q^{\frac{2}{3}} = 2, \text{ whence } q = 1 \text{ or } -8.$$

Given expression

$$= \frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{2q - (p^2 - 2q)}$$

$$= \frac{p(p-2a)(p-2b)(p-2c)}{4q - p^2}$$

$$= \frac{p(p^3 - 4pq + 8r)}{p^2 - 4q}.$$

9. A vessel makes two runs on a measured mile, one with the tide in m minutes and one against the tide in n minutes. Find the speed of the vessel through the water, and the rate the tide was running at, assuming both to be uniform.

Let x = rate of vessel per hour;

" y = " tide "

$$\left. \begin{aligned} \frac{1}{x+y} &= \frac{m}{60} \\ \frac{1}{x-y} &= \frac{n}{60} \end{aligned} \right\} \therefore x = 30 \left(\frac{1}{m} + \frac{1}{n} \right).$$

$$y = 30 \left(\frac{1}{m} - \frac{1}{n} \right).$$

10. Five points, A, B, C, O and P , lie on a right line. The distances of A, B and C , measured from the point O , are a, b , and c ; their distances measured from the point P are x, y and z . Prove that whatever be the positions of the points O and P ,

$$x^2(b-c) + y^2(c-a) + z^2(a-b) + (b-c)(c-a)(a-b) = 0.$$

We have

$$x = a + a, y = b + a, z = c + a, a = OP.$$

Proposed equation holds if

$$(a+a)^2(b-c) + (b+a)^2(c-a) + (c+a)^2(a-b) + (b-c)(c-a)(a-b) = 0,$$

if (since coeff'ts of powers of a cancel)

$$a^2(b-c) + \dots + (b-c)(c-a)(a-b) = 0$$

if $0 = 0$.

UNIVERSITY OF TORONTO.

EXAMINATION PAPERS: 1880.

FIRST EXAMINATION AND PRIMARY.

PROF. PIKE'S CHEMISTRY PAPERS IN MEDICINE.

Pass and Honours.

1. State the laws of the action of heat and pressure on gases. What volume will 100^{cc} of gas measured at 10^{°C} and under a pressure of 300^{mm} of mercury occupy at 19^{°C} and 762^{mm} of mercury?

2. How may each of the oxides of carbon be converted into the other? Calculate the percentage composition of carbon monoxide and of carbon dioxide.

3. Write equations representing the action of heat on the following substances: Potassium Chlorate, Ammonium Bichromate, Ammonium Nitrite, Ammonium Nitrate, Manganese Dioxide, Lead Nitrate, Phosphorous Acid.

4. Boron Chloride and Stannic Chloride are said to contain respectively three and