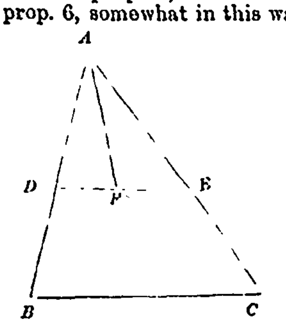
though E, F be so with respect to Q); and complete the triangles CAD. EBF. Then by (b) these are equal, since AP and BQ being perpendicular to CD, EF they are the altitudes. But the bases CD, EF are not in one line, except AP and BQ be parallel, which again can only be the case (retaining the common magnitude of the equal altitudes), when AP, BQ are perpendicular to the line AB.

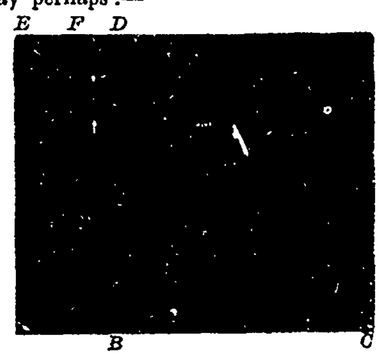
This shows that the inference of the truth of the converse from a principal theorem is in one case unsound; and it follows, that if a single exception to such a rule of inference can be produced, there may be more (it would be easy to produce any number demanded) than one; and hence again that any specific inference of the kind which we may wish to draw may be of these erroneous ones. In all cases, therefore, where such a proposition is required, it must be itself actually proved prior to its adoption.

It may be remarked too, though somewhat casually, that when the converse theorem admits of direct demonstration, the process itself really becomes identical with the analysis of the primary theorem The relation, however, between direct and indirect demonstrations in connection with analysis will be better seen when we come to that subject. We only give one or two examples here.

1. Take vi. 2 as a primary theorem, and one of its converses is given and proved. But there also arises this:

Let ABD be one line, and DE parallel to BC, but instead of taking AEC as one line, let $BA:BC::DA:DE$; then A, E, C, will be one line. It is true, indeed, that Euclid would not have proved it at this stage; but it is certain that if he had wanted it for any ulterior purpose, he would have enunciated and proved it after prop. 6, somewhat in this way perhaps:—





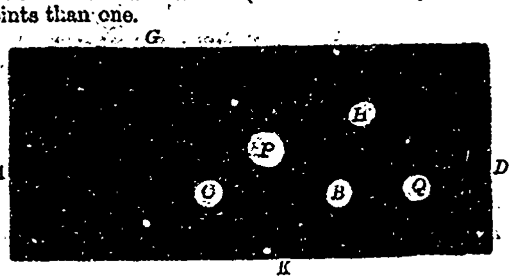
For if AC does not contain the point E let it cut DE in some other point F. Then line DE is parallel to BC the angle ADE is equal to ABC; and since the sides about the equal angles are proportional, the triangles BAC, DAE are equiangular; and the angle DAE is equal to BAC:—the less to the greater (or the greater to the less), which is absurd. The line AC cannot therefore but pass through E; or A, E, C are in one line.

Or thus, perhaps:—

For if not, draw as before. Then since AFC is a straight line, we have— $AD:DF::AB:BC$ and $AD:DE::AB:BC$, wherefore $AD:DF::AD:DE$, or (v. 7) $DF=DE$, the greater to the less, &c.

2. Another theorem of great importance in geometrical demonstration, takes one part of the hypothesis (viz., three lines drawn through the same point) in exchange for some conclusion respecting lines so drawn. Now that three lines which are specified in the enunciation in dependence upon other conditions should be proved to meet in one point, is incapable of direct proof proceeding wholly from first principles. It has been supposed that this difficulty is overcome by the process of showing that two of them divide the third in the same ratio, but the difficulty is only transformed into another.

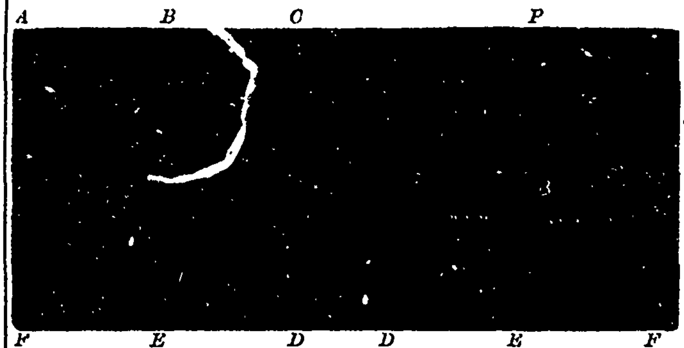
A given straight line can be divided in a given ratio, *the segments being estimated in the same manner* (a limitation always necessary) in more points than one.



Let AB be the given line, and $AG:BK$ the given ratio. Place AG, BK parallel to one another and on opposite sides of AB; draw GK cutting AB in D. Then $AC:CB::AG:BK$ in the given ratio. There can hence be one point of section.

There can be no other; for if possible let also $AP:PB::AG:BK$. Wherefore also $AP.PB::AC.CB$; and hence $AP+PB:AP::AC+CB.CB$, or $AP+PB.AC+BC::AP:CB$. But the first term is equal to the second, each being equal to AB; and hence the third to the fourth, or AP the greater equal to AC the less. Wherefore it cannot be divided, the homologous parts taken in the same manner in more points than one.

3. There is one case more of very frequent occurrence that must be noticed:—



If parallel lines, AC and FD, be divided in the same ratio, viz., $AB:BC::DE:EF$, the lines AD, BE, CF will pass through one point P. The same is true however many segments there be in these parallels. It is left for the student to prove.

In the same way, it may be shown that subject to the homologous limitation, the line can be divided *externally* in one point D in the given ratio $AG:BH$. Let the student prove it by assuming Q as another point.

As the character of these propositions will now be apparent, it will only be necessary to add one example of the direct proof of a converse proposition. It is the first of that remarkable series given in 1763 by Dr. Matthew Stewart under the modest title of *Propositiones Geometricæ, more veterum demonstratæ*; and is likewise made the first of those appropriated from that work (without acknowledgment) by Lawson in his *Dissertation on the Geometrical Analysis of the Antients*, 1774. The demonstration here employed was given in—in Legborn's *Mathematical Repository*, vol. I. —, by Mr. Colin Campbell, and reprinted amongst other works of that eminent geometer at Liverpool, 1848, under the title of *Mathematical Lucubrations.* Dr. Stewart's is altogether different, and it was no part of Lawson's plan to give solutions.

Directly.—"If a right line AB be bisected in E, and two points C and D be taken in it, such that $AC:CB::AD:DB$, the rectangle DCE will be equal to the rectangle ACB."

Conversely.—"If a right line AB be bisected in E and two points C and D be taken in it, such that the rectangles DCE, ACB be equal, then $AC:CB::AD:DB$."

Directly—

Since	$AC:CB::AD:DB$,
inv.	$CB:CA::DB:AD$,
div. or comp.	$2CE:AC::AB:AD$,
hence	$CE:CA::BE:AD$,
perm.	$CE:BE::AC:AD$,
inv.	$BE:CE::AD:AC$,
comp.	$CB:CE::DC:AC$,
wherefore rect.	DCE=rect. ACB.

Conversely—(Same figure.)

Because rect.	DCE=rect. ACB,
it will be	$DC:AC::CB:CE$,
div.	$AD:AC::EB:EC$,
perm.	$AD:EB::AC:CE$,
or	$AD:AB::AC:2CE$,
comp. or div.	$AD:BD::AC:CB$,
that is	$AC:CB::AD:DB$.

Instead of giving a series of theorems of this class, it is left to the tutor to select such as in his judgment are best adapted to his pupils. One thing, however, every tutor should insist upon—that whenever his pupil employs or quotes the converse of any specific theorem, he should require a proof to be given, direct or indirect as the case may admit.