

7. Simplify  $(a+b+c)^2 + (a+b-c)^2 + (a+c-b)^2 + (b+c-a)^2$ .

See number 3.  $a^2 + a^2 + a^2 + a^2 = 4a^2$   
 $2ab + 2ab - 2ab - 2ab = 0$ .  
 $\therefore$  Answer  $= 4(a^2 + b^2 + c^2)$ .

8. Simplify  $(ax+by+cz)^2 + (ax+cy+bz)^2 + (bx+ay+cz)^2$   
 $+ (bx+cy+az)^2 + (cx+ay+bz)^2 + (cx+by+az)^2$ .

[N.B.—Observe the symmetry. Only  $a, b, c$  are permuted. If we write  $a, b, c$  in circular order, thus  $c, a, b$ , starting with  $a$ , we

may read off  $a, b, c$  and  $a, c, b$ , the coefficients of the first two terms. Similarly, starting from  $b$  and from  $c$ , we can read off the other four sets of coefficients.  $Cx$ , notice that if we change  $a$  into  $b$ ,  $b$  into  $c$ , and  $c$  into  $a$ , the first term and the fifth change places, also the second and the third, and the fifth and the sixth, so that the expression remains as at first. This is the proof of symmetry, which in the present example was patent enough without testing, though it does not always happen to be so manifest by mere inspection.]

Looking at the perfect squares, and also at the double products, we see that  $2a^2(x^2+y^2+z^2) + 4ab(xy+yz+zx)$  is part of the result. Hence by symmetry

$$2(a^2+b^2+c^2)(x^2+y^2+z^2) + 4(ab+bc+ca)(xy+yz+zx)$$

is the whole result, for the sum must consist wholly of squares and double products.

9. Simplify  $(a+b+c)^2 + (a+b-c)^2 + (a+c-b)^2 + (b+c-a)^2$ .

[N.B.—First observe the symmetry. The signs only being permuted.]

Reasoning as above, we see that  $a^3 + a^3 + a^3 = 3a^3$ , i.e.,  $2a^3$  is part of the result, and also  $3a^2b + 3a^2b - 3a^2b + 3a^2b$ , i.e.,  $6a^2b$  is part of the result.

Now, perfect cubes have only one other sort of term, viz.,  $abc$ .

Therefore,  $2(a^3+b^3+c^3) + 6(a^2b+a^2c+b^2a+b^2c+c^2a+c^2b)$  is part of it. To find its coefficients, put  $a=b=c=1$ , when given expression  $= 30$ . This shows that the expansion contains 30 terms; but the part already found gives 42 terms, hence the remaining part is  $-12abc$ .

$$\text{Ans.} = 2(a^3+b^3+c^3) + 6(a^2b+a^2c+b^2a+b^2c+c^2a+c^2b) - 12abc.$$

10. If  $x$  is an odd number  $x^2 - x$  is divisible by 24, and

$$(x^2+3)(x^2+7) \text{ by } 32.$$

(a) For  $x^2 - x = (x-1)x(x+1)(x^2+1)$ . Also, since  $x$  is odd it is of the form  $2n+1$ . Substitute this for  $x$  and we have

$$(2n)(2n+1)(2n+2)(4n^2+4n+2).$$

Now the first three factors are consecutive numbers, and some one of them must be divisible by 3. It is also plain that 8 is a factor of the expression,  $\therefore 24$  must be a factor.

(b) Substitute  $2n+1$  for the odd number  $x$  and we have

$$16(n^2+n+1)(n^2+n+1),$$

and the last two factors are two consecutive numbers,  $\therefore$  one of them must be even, and expression  $=$  an even multiple of 16, i.e., a multiple of 32.

11. If  $4a^2b^2c^2(x^2+y^2+z^2)(a^2x^2+b^2y^2+c^2z^2)$

$$= \{(b^2+c^2)a^2x^2 + (c^2+a^2)b^2y^2 + (a^2+b^2)c^2z^2\}^2$$

when  $a > b$ , and  $b > c$ , show that  $y = 0$ .

Multiply out, and arrange in powers of  $y$

$$(a^2-c^2)b^4y^4 + 2\{(a^2-c^2)(b^2-c^2)a^2x^2 + (a^2-c^2)(a^2-b^2)c^2z^2\}b^2y^2 + \{(b^2-c^2)a^2x^2 - (a^2-b^2)c^2z^2\} = 0.$$

Now  $a^2 > b^2 > c^2$ ,  $\therefore a^2 - c^2$ ,  $b^2 - c^2$ ,  $a^2 - b^2$ , are all positive quantities.

$\therefore$  the coefficients of  $y^4$  and  $y^2$  are positive.

$\therefore$  it is necessary that each term  $= 0$ , if their sum  $= 0$  or  $y = 0$ .

12. Given  $x+y+z=0$ ;  $x_1+y_1+z_1=0$ , show that

$$(x^2+x_1^2)yz + (y^2+y_1^2)zx + (z^2+z_1^2)xy$$

$$= (x+x_1)y_1z_1 + (y+y_1)z_1x_1 + (z+z_1)x_1y_1.$$

Multiply out and  $xyz(x+y+z) + x_1^2yz + y_1^2zx + z_1^2xy$

$$= x_1y_1z_1(x_1+y_1+z_1) + x^2y_1z_1 + y^2z_1x_1 + z^2x_1y_1,$$

$$\text{i. e., } x_1^2yz + y_1^2zx + (x_1+y_1)^2xy = x^2y_1z_1 + y^2z_1x_1 + (x+y)^2x_1y_1,$$

or,  $x_1^2y(z+x) + y^2x(z+y) = x^2y_1(z_1+x_1) + y^2x_1(z_1+y_1)$ , an identity on multiplying out.

#### PROBLEMS FOR SOLUTION.

By T. F. O. Penotanguishene. A point is taken in an equilateral triangle, and the distances from that point to the angles are respectively 10,  $7\frac{1}{2}$ , and  $12\frac{1}{2}$  chains. Find the area of the triangle.

By T. F. O., Appin, Ont. 1. A mortgage of \$3000 is drawn for nine years @ 7%. The principal is payable in equal annual instalments, and interest on all unpaid principal payable therewith. How much must a man pay for this mortgage in order to realise 8% per annum, on his money?

2. A man and a boy work at a job on alternate days. The boy can do it alone in thirteen days. If the man begin first the work will be completed half a day sooner than it would be were the boy to begin first. Find in what time both man and boy working together can do the work.

3. Is the answer given correct—Smith and McMurphy's *Advances Arithmetic*. No 20 p. 264?

For the benefit of readers we append the problem and answer given. Editor Math. Dep.

A semicircular plot of ground whose radius is 12 yds. has inside the circumference a path 2 yds. wide; the rest of the space is a flower-bed. Find the size of the bed. Answer 100 sq. yds. 5 sq. ft. 20 sq. in.

#### Contributions.

##### HENRY WADSWORTH LONGFELLOW,

The death of Longfellow leaves a blank in the roll of American litterateurs that will not be easily refilled. If not the greatest of American poets he is at least fairly entitled to the post of pre-eminence amongst the poets whom America has produced. In one sense, and that a very important one, he is hardly an American poet at all, for his modes of treating his subjects, and very often his subjects themselves, belong to the Old World rather than the New. It seems strange that a man of his fine intellect and generous sympathies could live from 1807 to 1882 and witness the progress made by his own country through its turbulent struggles upward to higher national life without showing in his writings some traces of the effect produced by those struggles on himself—strange, but not unprecedented, for the Sturm-und-Drang period of German national life had just as little effect on the placid temperament of Goethe. When Longfellow did choose an American subject it was usually legendary in its character and as remote as it well could be from topics suggested by the surging democracy by which he was surrounded but of which he himself formed no part, and the real character of which he apparently never understood.

Longfellow is one of the poets of nature, and of nature in her calmer moods. He loves the sunshine and the zephyr, not the thunder-cloud or the hurricane, just as he prefers to depict humanity in comfort and at rest rather than humanity panting and struggling to free itself from the fetters of evil that hamper and irritate it. The best passages in his best poems are those which delineate with loving minuteness of detail the impressions produced by nature, and the nearer her condition to one of perfect repose the more fondly he dwells upon the scene he is depicting. Of all his poems "Evangeline" furnishes the best illustration of this peculiarity and nothing even in "Evangeline" can surpass the following brief description of the Indian Summer:—

Such was the advent of autumn. Then followed the beautiful season  
 Called by the pious Acadian peasants the Summer of All-Saints!  
 Filled was the air with a dreamy and magical light, and the landscape  
 Lay as if new-created in all the freshness of childhood.  
 Peace seemed to reign upon Earth, and the restless heart of the Ocean  
 Was for a moment consoled.

Longfellow's highest claim to approbation is the absolute purity of his life and writings. Not an incident of the former known to