by induction we find that we may choose r things out of n things in

$n(n-1)(n-2)\ldots(n-r+1)$ ways

Now $n(n-1)(n-2)\ldots(n-r+1)$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)(n-r)\ldots 3.2.1.}{1, 2, 3\ldots n-r}$$

$$= \frac{\lfloor n}{\lfloor n r} \quad \text{or} \quad \frac{n!}{(n-r)!}$$

But if ${}^nC_r$ be the number of combinations of n things r together, each combination may give rise to $r!$ permutations.

$$\therefore {}^nC_r \times r! = \frac{n!}{(n-r)!}$$

$$\therefore {}^nC_r = \frac{n!}{r!(n-r)!}$$

(*b*). $n-p$ lines not concurrent determine $\frac{1}{2}(n-p)(n-p-1)$ points. The p lines can interest the n-p lines in $p(n-p)$ points altogether. Besides this p lines determine one point. Therefore the number of points determined is, $\frac{1}{2}(n-p)(n-p-1) + (n-p)p+1$ or, $\frac{1}{2}(n-p)(n+p-1)+1$

9. (*a*) Write the r^{th} term in the expansion of $(1-x)^{-n}$, and state what this coefficient is in regard to homogeneous products.

(*b*) If p, q, s denote the product, quotient, and sum, respectively, of two numbers, then

$p=s^2 \{ q-2q^2+3q^3- + \ldots \text{ad inf.} \}$

9. (*a*). $(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{1\ 12}x^2 - + \ldots$ and the coeff of x^r – is

$$\frac{n(n+1)(n+2)\ldots(n+r-1)}{r!}$$

This is an expression for the number of homogeneous terms of r dimensions that can be made from n letter and their powers.

(*b*). Let x, y denote two numbers.

Then $p = xy$, $s = x+y$, $q = \frac{y}{x}$ from which we must elininate x and y

Thus, $y = \frac{p}{x} = s - x = qx$,

Whence $\frac{p}{q} = x^2$ and $x = \frac{s}{1+q}$

$\therefore \frac{p}{q} = \frac{s^2}{1+q^2} = s^2(1-2q+3q^2-+\ldots\}$

10. (*a*) Find expressions for the sum of an A.P. and of a G.P.

(*b*) If the n^{th} terms of two A.P.s be respectively $a-bn$ and $b+an$, show that they have a common sum for the same number of terms if $\frac{4b}{a+b}$ is a negative integer; and that under the same circumstance they have also a common n^{th} term.

10. (*a*) This is common book-work.

b. The series are: $a-b$, $a-2b$, $b+3a$, etc., and their sums are: $na+\frac{1}{2}n(n+1)b$, and $nb+\frac{1}{2}n(n+1)a$, and these being equal gives $n = \frac{a-3b}{a+b} = \frac{a+b-4b}{a+b} = 1 - \frac{4b}{a+b}$.

$\because$ If $\frac{4b}{a+b}$ is a negative integer, n is a positive whole number.

Also they have a common n th. term if $a-bn=b+an$, i.e. if $n=1-\frac{2b}{a+b}$.

But if $\frac{2b}{a+b}$ is a negative integer, so also $\frac{4b}{a+b}$.

11. (*a*) Establish a formula for the present worth of A dollars due in t years at r cents per dollar compound interest.

(*b*). A man 30 years old enjoys $600 annually from the rent of buildings, during his life. If his probable duration of life is 20 years more, what is the present cash value of his anuity, money being at r cents on the dollar?

(Only a symbolic result is required.)

11. (*a*) Let P denote the present worth, then P put to interest for t years at r per unit should give A.

$$\therefore A=P)1+r)^t.$$

$$\therefore P=\frac{A}{(1+r)^t}.$$

(*b*). Let P be the present cost value and et A denote the annual income.

The last payment is A.

The payment before the last is AR, when $R=1+r$. The next previous is AR^2, etc. So that the total amount of all the payments at the end of 20 years is:

$$A(1+R+R^2+\ldots R^{19})=A\frac{R^{20}-1}{R-1}.$$

But P put to interest for 20 years should give this sum:

$$\therefore P(1+r)^{20}=A.\frac{R^{20}-1}{R-1};$$

$$\text{or, } P=A.\frac{R^{20}-1}{R^{20}(R-1)}.$$