ARTS DEPARTMENT.

ARCHIBALD MACMURCHY, M.A., MATHEMATICAL EDITOR, *C. E. M.*

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SOLUTION

by proposer, Professor EDGAR FRISBY, M.A., Naval Observatory, Washington.

168. Find the value of x and y in the following equations.

$$x^2-y^2=a^2 \qquad (1).$$
$$x^3+3xy^2=b^3 \qquad (2).$$
$$x^6-3x^4y^2+3x^2y^4-y^6=a^6=(1)^3.$$
$$x^6+6x^4y^2+9x^2y^4=b^6=(2)^2.$$
$$\therefore\ y^2(9x^4+6x^2y^2+y^4)=b^6-a^6.$$
$$\therefore\ 3x^2y+y^3=\pm\sqrt{b^6-a^6} \qquad (3).$$
$$\therefore\ x+y=\left\{b^3\pm\sqrt{b^6-a^6}\right\}^{\frac{1}{3}}$$
$$=\sqrt[3]{(2)+(3)}=(4).$$
$$x-y=\frac{a^2}{\left\{b^3\pm\sqrt{b^6-a^6}\right\}^{\frac{1}{3}}}=(5).$$
$$x=\tfrac{1}{2}\left\{(4)+(5)\right\},\quad y=\tfrac{1}{2}\left\{(4)-(5)\right\}.$$

Solutions to Problems in December and January numbers, by the proposer, J. L. Cox, B.A., Math. Master, Collegiate Institute, Collingwood.

187. If the sides of a triangle be cut proportionally and lines be drawn from the points of section to the opposite angles, the intersection of these lines will be in the same line, viz., that drawn from the vertex to the middle of the base.

Let the sides AB, AC of the triangle ABC be cut proportionally in D, F, H; E, G, L respectively, so that

$$AD:AE::DF:EG::FH:GL::HB:LC.$$

Join BE, BG, BL, CD, CF, CH; these lines will intersect each other in the line AK, drawn from A to K, the middle point of BC.

Join DE. Then since when any number of magnitudes are proportional, as one antecedent is to its consequent, so are all the antecedents to all the consequents;

$\therefore$ $AD:AE::AB:AC$, and DE is parallel to BC. Join KO (BE and CD intersect at O), and let it meet DE in I. The triangles BOK, IOE are similar, and

$$\therefore\ BK:KO::EI:IO,$$

and for same reason

$$CK:KO::DI:IO;$$

$\therefore$ $EI=DI$, and DE is bisected by KO, and it also is bisected by AK; $\therefore$ AK passes through O. In same manner it may be shewn that BG and CF, as also BL and CH intersect each other in points which are in the line AK.

188. Given the base and perpendicular: to construct the triangle, when the rectangle contained by the sides is equal to twice the rectangle contained by the segments of the base made by the line bisecting the vertical angle.

Let AB be equal to the given base, and draw the indefinite line ED bisecting it at right angles at point C. Take CE, CD each equal to the given perpendicular, and through the points A, D, B describe circle. Draw EF parallel to AB, meeting the circle in F. Join AF, FB; AFB is the triangle required. Draw FG perpendicular to AB; it is equal to the given perpendicular. Join FD meeting AB in H; since DC is equal to CE;