The deflection at the centre for the same load P' is

$$y' = \frac{P'l^3}{48EI}$$

It follows that

$$u = v\,\frac{(3l^2x - 4x^3)}{l^3}$$

We are now in a position to evaluate two of the integrals.

$$\int \frac{\delta W_b}{2g}\,u^2 = \frac{W_b v^2}{g l^7}\int_0^{\frac{l}{2}}(3l^2x - 4x^3)^2\,dx$$

$$= \frac{17}{35}\,\frac{W_b}{2g}\,v^2$$

Also

$$\int \delta W_b z = \frac{2W_bP}{48EIl}\int_0^{\frac{l}{2}}(3l^2x - 4x^3)\,dx$$

$$= \frac{5}{8}W_b y \qquad \text{since } \frac{1}{48}\,\frac{Pl^3}{EI} = y$$

Inasmuch as the average weight of the beam is only four pounds and the work done by gravity in deflecting it is only $\frac{5}{8} \times 4 \times y$, while the work done by the descending tup will average $800y$, the error introduced by omitting this integral is also very small. Moreover, since one of the purposes of the discussions is to make a comparison between the energy of rupture for static and impact bending, and the former test disregards the work done by gravity on the beam, it will also be neglected here.

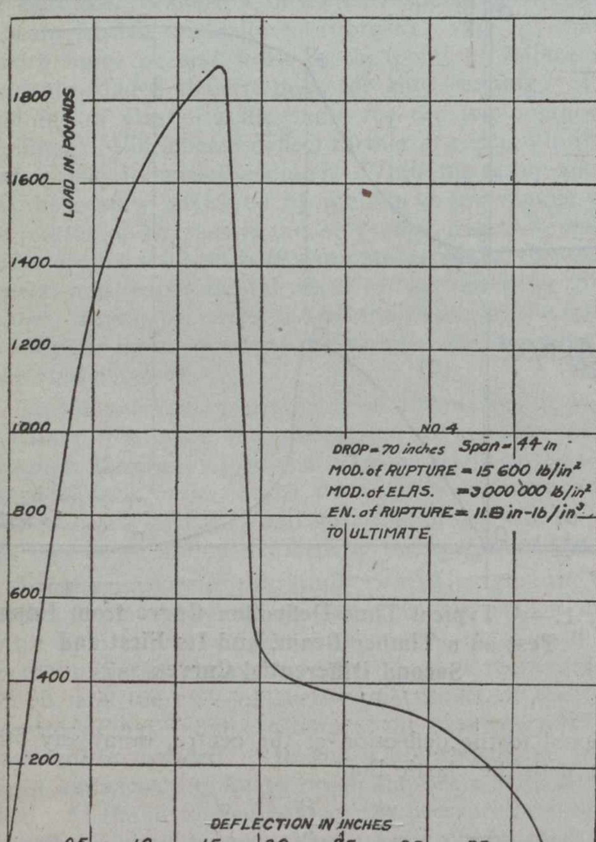

Fig. 2.—Impact Force Deflection Curve for a Long-leaf Yellow Pine Beam Completely Ruptured by a Single Blow

There results, then, upon replacing in equation (2) the integrals valuated, the new equation

$$\int F dy + \frac{17}{35}\,\frac{W_b}{2g}v^2 + \frac{1}{2}\,\frac{W_t}{g}v^2 = W_t y + \frac{1}{2}\,\frac{W_t}{g}v_t^2$$

For any one height of drop v_t is a constant, hence on differentiating

$$\frac{F dy}{dt} + \left(W_t + \frac{17}{35}W_b\right)\frac{v}{g}\,\frac{dv}{dt} = W_t\frac{dy}{dt}$$

But

$$\frac{dy}{dt} = v' \qquad \text{and} \quad \frac{dv}{dt} = \frac{d^2y}{dt^2} = \frac{d^2s}{dt^{2\prime}} \quad \text{so that}$$

$$Fv + \left(W_t + \frac{17}{35}W_b\right)\frac{v}{g}\,\frac{d^2s}{d\,t} = W_t v$$

$$F = W_t - \left(W_t + \frac{17}{35}W_b\right)\frac{1}{g}\,\frac{d^2s}{dt^2} \qquad (3)$$

From the last equation it is seen that the beam must exert a force upward that will overcome the weight of the tup and impart to a mass of $\left(W_t + \frac{17}{35}W_t\right)$ pounds an acceleration $\frac{d^2s}{dt^2}$.

The latter should be noted in particular, for it shows that the beam imparts an acceleration not only to the tup but to forces in the beam also. This is obvious when one remembers that at the instant when the inertia of the beam has been overcome, which happens very soon after initial contact, the beam has acquired a considerable velocity, and this velocity is reduced to almost zero at the time of maximum deflection. These forces proceed, of course, from the elastic deformation of the fibres.

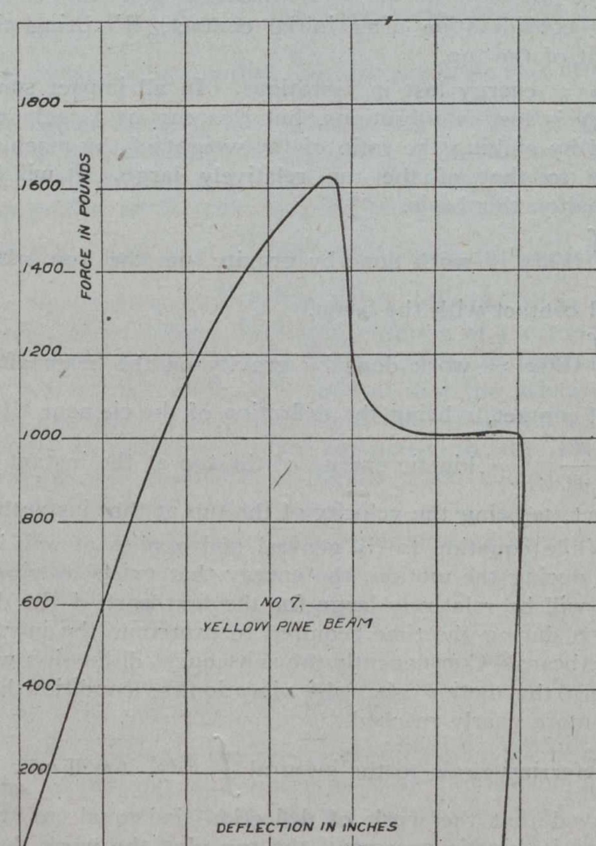

Fig. 3.—Impact Force-Deflection Curve for a Long-leaf Yellow Pine Beam Fractured but Not Entirely Ruptured

The force F being a centre force acting on a simple beam supported at the ends, the external or bending moment is obtained immediately, and equating to the internal or resisting moment there results.