

ing definitions, and introducing the manner of proof which has ever since been in use." Let us carefully remember that "one chief characteristic of the mathematical work of Pythagoras was the combination of arithmetic with geometry," culminating in the theory of proportion. "In this respect he is fully comparable to Descartes, to whom we owe the decisive combination of algebra with geometry." Allman says of this unifying aspect of his work: "We are plainly in presence of, not merely a great mathematician, but of a great philosopher. It has ever been so; the greatest steps in the development of mathematics have been made by philosophers."

Of equal importance with the question of the historical order of development of the *matter* of geometrical knowledge is a consideration of the attitude of mind of the ancients towards the subject, the spirit in which at different times they cultivated geometry, as art or science or both. First we find the Egyptians employing a crude empirical geometry for architecture and land-surveying, rendered necessary by the obliteration of landmarks caused by Nile floods. These approximate rules of thumb come to the knowledge of a people of higher intellectual calibre — the Greeks. Hence there gradually emerges the vague conception of the possibility of a *science* of geometry, in which clear, abstract definitions shall refine on mere sense-perceptions, axioms peculiar to geometry combine with axioms at the base of all reasoning, and thereby the empirical laws be absorbed once for all in rigorously deduced abstract theorems. Of course the emergence of all this was very gradual; *there was incessant action and reaction between the concrete and the abstract* (a fact of fundamental importance for education, remark). At length we reach a time when geometrical knowledge has assumed a perfectly abstract form, become evolved into a science; we find it now in the hands of professional philosophers, who follow and value the study of it partly as an intellectual discipline, and partly out of scientific curiosity, but with no other motive. Plato (himself a student of geometry, though apparently not a specialist therein) appears simply to express a feeling common in his time when he denounces the application of scientific geometry to "vulgar handicraft" as demeaning to the science; and we all know the motto written over the entrance to his Academy: "Let none ignorant of geometry enter my door." To Plato and his attitude I shall presently return.

This divorce of geometrical science from the needs of common life must not be misinterpreted as a sundering of the abstract from the concrete; bearing in mind the presumed educational application of this epitome of the history of geometry, I lay great stress on the fact that, "side by side with the development of abstract geometry by the Greeks, the *practical art of geometrical drawing*, which they derived originally from the Egyptians, continued to be in use." The true significance of this must not be overlooked.

The ideal of Greek geometry may fairly be described as *construction under self-imposed definite limitations*. For example, as regards problems in a plane, from the abstract side of thought the attempt was made to solve all such by ultimate reference to the concepts, straight line and circle; from the concrete standpoint, all constructions were to be reduced to use of ruler and compasses only (the respective concrete embodiments of the ideal straight line and circle). In the former aspect geometry was entirely independent of mechanics, but in the latter dependent on it; but not for long can the two be separated without gravest danger to arrestment of the one as art and of